pytorch

Lecture 22

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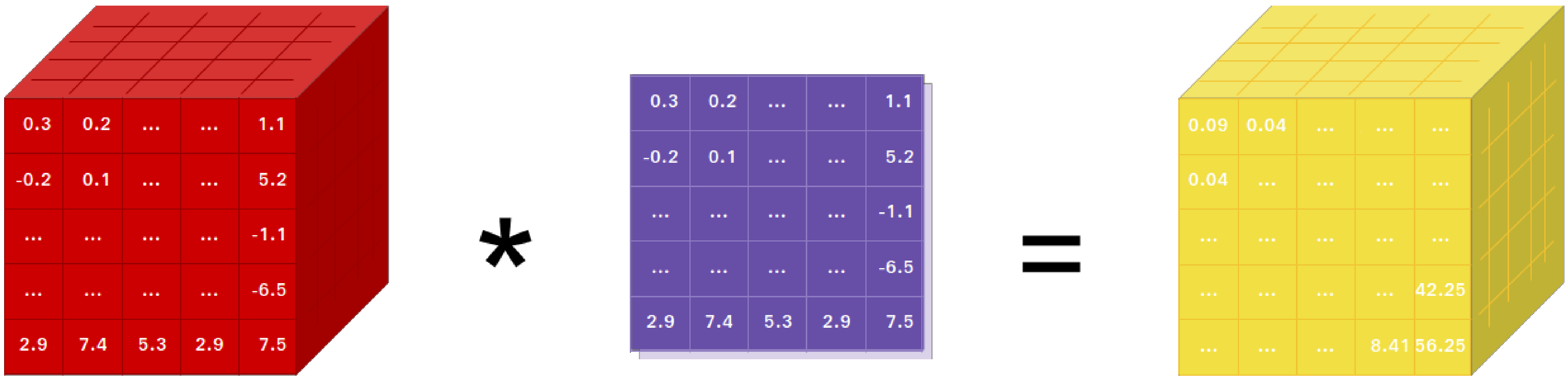
PyTorch

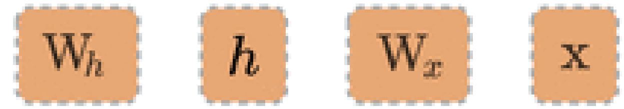
> PyTorch is a Python package that provides two high-level features:
>
> - Tensor computation (like NumPy) with strong GPU acceleration
> - Deep neural networks built on a tape-based autograd system




```python
import torch
torch.__version__
```

```
'2.0.0'
```

A graph is created on the fly




```
W_h = torch.randn(20, 20, requires_grad=True)
W_x = torch.randn(20, 10, requires_grad=True)
x = torch.randn(1, 10)
prev_h = torch.randn(1, 20)
```

Tensors

are the basic data abstraction in PyTorch and are implemented by the `torch.Tensor` class. The behave in much the same was as the other array libraries we've seen so far (`numpy`, `theano`, etc.)

```python
torch.zeros(3)
```

```
tensor([0., 0., 0.])
```

```python
torch.ones(3,2)
```

```
tensor([[1., 1.],
        [1., 1.],
        [1., 1.]])
```

```python
torch.empty(2,2,2)
```

```
tensor([[[0., 0.],
         [0., 0.]],

        [[0., 0.],
         [0., 0.]]])
```

```python
torch.manual_seed(1234)
```

```
<torch._C.Generator object at 0x29951c510>
```

```python
torch.rand(2,2,2,2)
```

```
tensor([[[[0.0290, 0.4019],
          [0.2598, 0.3666]],

         [[0.0583, 0.7006],
          [0.0518, 0.4681]]],


        [[[0.6738, 0.3315],
          [0.7837, 0.5631]],

         [[0.7749, 0.8208],
          [0.2793, 0.6817]]]])
```

Constants

As expected, tensors can be constructed from constant numeric values in lists or tuples.

```
torch.tensor(1)
```

```
tensor(1)
```

```
torch.tensor((1,2))
```

```
tensor([1, 2])
```

```
torch.tensor([[1,2,3], [4,5,6]])
```

```
tensor([[1, 2, 3],
        [4, 5, 6]])
```

```
torch.tensor([(1,2,3), [4,5,6]])
```

```
tensor([[1, 2, 3],
        [4, 5, 6]])
```

```
torch.tensor([(1,1,1), [4,5]])
```

```
Error: ValueError: expected sequence of length 3 at d
```

```
torch.tensor([["A"]])
```

```
Error: ValueError: too many dimensions 'str'
```

```
torch.tensor([[True]]).dtype
```

```
torch.bool
```

Note using `tensor()` in this way results in a full copy of the data.

Tensor Types

Data type	`dtype`	`type()`	Comment
32-bit float	`float32` or `float`	`FloatTensor`	Default float
64-bit float	`float64` or `double`	`DoubleTensor`	
16-bit float	`float16` or `half`	`HalfTensor`	
16-bit brain float	`bfloat16`	`BFloat16Tensor`	
64-bit complex float	`complex64`		
128-bit complex float	`complex128` or `cdouble`		
8-bit integer (unsigned)	`uint8`	`ByteTensor`	
8-bit integer (signed)	`int8`	`CharTensor`	
16-bit integer (signed)	`int16` or `short`	`ShortTensor`	
32-bit integer (signed)	`int32` or `int`	`IntTensor`	
64-bit integer (signed)	`int64` or `long`	`LongTensor`	Default integer
Boolean	`bool`	`BoolTensor`	

We've left off quantized integer types here

Specifying types

Just like NumPy and Pandas, types are specified via the `dtype` argument and can be inspected via the dtype attribute.

```
a = torch.tensor([1,2,3]); a
```

```
tensor([1, 2, 3])
```

```
a.dtype
```

```
torch.int64
```

```
b = torch.tensor([1,2,3], dtype=torch.float16);
```

```
tensor([1., 2., 3.], dtype=torch.float16)
```

```
b.dtype
```

```
torch.float16
```

```
c = torch.tensor([1.,2.,3.]); c
```

```
tensor([1., 2., 3.])
```

```
c.dtype
```

```
torch.float32
```

```
d = torch.tensor([1,2,3], dtype=torch.float64);
```

```
tensor([1., 2., 3.], dtype=torch.float64)
```

```
d.dtype
```

```
torch.float64
```


Note the default types are slightly different from the other array/tensor libraries

Type precision

When using types with less precision it is important to be careful about underflow and overflow (ints) and rounding errors (floats).

```
torch.tensor([128], dtype=torch.int8)
```

```
Error: RuntimeError: value cannot be converted to ty
```

```
torch.tensor([128]).to(torch.int8)
```

```
tensor([-128], dtype=torch.int8)
```

```
torch.tensor([255]).to(torch.uint8)
```

```
tensor([255], dtype=torch.uint8)
```

```
torch.tensor([300]).to(torch.uint8)
```

```
tensor([44], dtype=torch.uint8)
```

```
torch.tensor([300]).to(torch.int16)
```

```
tensor([300], dtype=torch.int16)
```

```
torch.tensor(1/3, dtype=torch.float16)
```

```
tensor(0.33325195, dtype=torch.float16)
```

```
torch.tensor(1/3, dtype=torch.float32)
```

```
tensor(0.33333334)
```

```
torch.tensor(1/3, dtype=torch.float64)
```

```
tensor(0.33333333, dtype=torch.float64)
```

```
torch.tensor(1/3, dtype=torch.bfloat16)
```

```
tensor(0.33398438, dtype=torch.bfloat16)
```

NumPy conversion

It is possible to easily move between NumPy arrays and Tensors via the `from_numpy()` function and `numpy()` method.

```
a = np.eye(3,3)
torch.from_numpy(a)
```

```
tensor([[1., 0., 0.],
        [0., 1., 0.],
        [0., 0., 1.]], dtype=torch.float64)
```

```
b = np.array([1,2,3])
torch.from_numpy(b)
```

```
tensor([1, 2, 3])
```

```
c = torch.rand(2,3)
c.numpy()
```

```
array([[0.28367, 0.65673, 0.23876],
       [0.73128, 0.60122, 0.30433]], dtype=float32)
```

```
d = torch.ones(2,2, dtype=torch.int64)
d.numpy()
```

```
array([[1, 1],
       [1, 1]])
```

Math & Logic

Just like NumPy torch `tensor` objects support basic mathematical and logical operations with scalars and other tensors - torch provides implementations of most commonly needed mathematical functions.

```python
torch.ones(2,2) * 7 -1
```

```
tensor([[6., 6.],
        [6., 6.]])
```

```python
torch.ones(2,2) + torch.tensor([[1,2], [3,4]])
```

```
tensor([[2., 3.],
        [4., 5.]])
```

```python
2 ** torch.tensor([[1,2], [3,4]])
```

```
tensor([[ 2,  4],
        [ 8, 16]])
```

```python
2 ** torch.tensor([[1,2], [3,4]]) > 5
```

```
tensor([[False, False],
        [ True,  True]])
```

```python
x = torch.rand(2,2)

torch.ones(2,2) @ x
```

```
tensor([[1.22126317, 1.36931109],
        [1.22126317, 1.36931109]])
```

```python
torch.clamp(x*2-1, -0.5, 0.5)
```

```
tensor([[-0.49049568,  0.25872374],
        [ 0.50000000,  0.47989845]])
```

```python
torch.mean(x)
```

```
tensor(0.64764357)
```

```python
torch.sum(x)
```

```
tensor(2.59057426)
```

```python
torch.min(x)
```

```
tensor(0.25475216)
```

Broadcasting

Like NumPy in cases where tensor dimensions do not match, the broadcasting algorithm is used. The rules for broadcasting are:

- Each tensor must have at least one dimension - no empty tensors.
- Comparing the dimension sizes of the two tensors, going from last to first:
    - Each dimension must be equal, or
    - One of the dimensions must be of size 1, or
    - The dimension does not exist in one of the tensors

Exercise 1

Consider the following 6 tensors:

```
a = torch.rand(4, 3, 2)
b = torch.rand(3, 2)
c = torch.rand(2, 3)
d = torch.rand(0)
e = torch.rand(3, 1)
f = torch.rand(1, 2)
```

which of the above could be multiplied together and produce a valid result via broadcasting (e.g. `a*b`, `a*c`, `a*d`, etc.).

Explain why or why not broadcasting was able to be applied in each case.

Inplace modification

In instances where we need to conserve memory it is possible to apply many functions such that a new tensor is not created but the original value(s) are replaced. These functions share the same name with the original functions but have a `_` suffix.

```
a = torch.rand(2,2)
print(a)
```

```
tensor([[0.31861043, 0.29080772],
        [0.41960979, 0.37281448]])
```

```
print(torch.exp(a))
```

```
tensor([[1.37521553, 1.33750737],
        [1.52136779, 1.45181501]])
```

```
print(a)
```

```
tensor([[0.31861043, 0.29080772],
        [0.41960979, 0.37281448]])
```

```
print(torch.exp_(a))
```

```
tensor([[1.37521553, 1.33750737],
        [1.52136779, 1.45181501]])
```

```
print(a)
```

```
tensor([[1.37521553, 1.33750737],
        [1.52136779, 1.45181501]])
```

For functions without a `_` variant, check if they have a `to` argument which can be used instead - e.g. see

Inplace arithmetic

All arithmetic functions are available as methods of the Tensor class,

```
a = torch.ones(2, 2)
b = torch.rand(2, 2)
```

```
a+b
```

```
tensor([[1.37689185, 1.01077938],
        [1.94549370, 1.76611161]])
```

```
print(a)
```

```
tensor([[1., 1.],
        [1., 1.]])
```

```
print(b)
```

```
tensor([[0.37689191, 0.01077944],
        [0.94549364, 0.76611167]])
```

```
a.add_(b)
```

```
tensor([[1.37689185, 1.01077938],
        [1.94549370, 1.76611161]])
```

```
print(a)
```

```
tensor([[1.37689185, 1.01077938],
        [1.94549370, 1.76611161]])
```

```
print(b)
```

```
tensor([[0.37689191, 0.01077944],
        [0.94549364, 0.76611167]])
```

Changing tensor shapes

The `shape` of a tensor can be changed using the `view()` or `reshape()` methods. The former guarantees that the result shares data with the original object (but requires contiguity),the latter may or may not copy the data.

```
x = torch.zeros(3, 2)
y = x.view(2, 3)
y
```

```
tensor([[0., 0., 0.],
        [0., 0., 0.]])
```

```
x.fill_(1)
```

```
tensor([[1., 1.],
        [1., 1.],
        [1., 1.]])
```

```
y
```

```
tensor([[1., 1., 1.],
        [1., 1., 1.]])
```

```
x = torch.zeros(3, 2)
y = x.t()
z = y.view(6)
```

```
Error: RuntimeError: view size is not compatible witl
```

```
z = y.reshape(6)
x.fill_(1)
```

```
tensor([[1., 1.],
        [1., 1.],
        [1., 1.]])
```

```
y
```

```
tensor([[1., 1., 1.],
        [1., 1., 1.]])
```

```
z
```

```
tensor([0., 0., 0., 0., 0., 0.])
```

Adding or removing dimensions

The `squeeze()` and `unsqueeze()` methods can be used to remove or add length 1 dimension(s) to a tensor.

```
x = torch.zeros(1,3,1)
x.squeeze().shape
```

```
torch.Size([3])
```

```
x.squeeze(0).shape
```

```
torch.Size([3, 1])
```

```
x.squeeze(1).shape
```

```
torch.Size([1, 3, 1])
```

```
x.squeeze(2).shape
```

```
torch.Size([1, 3])
```

```
x = torch.zeros(3,2)
x.unsqueeze(0).shape
```

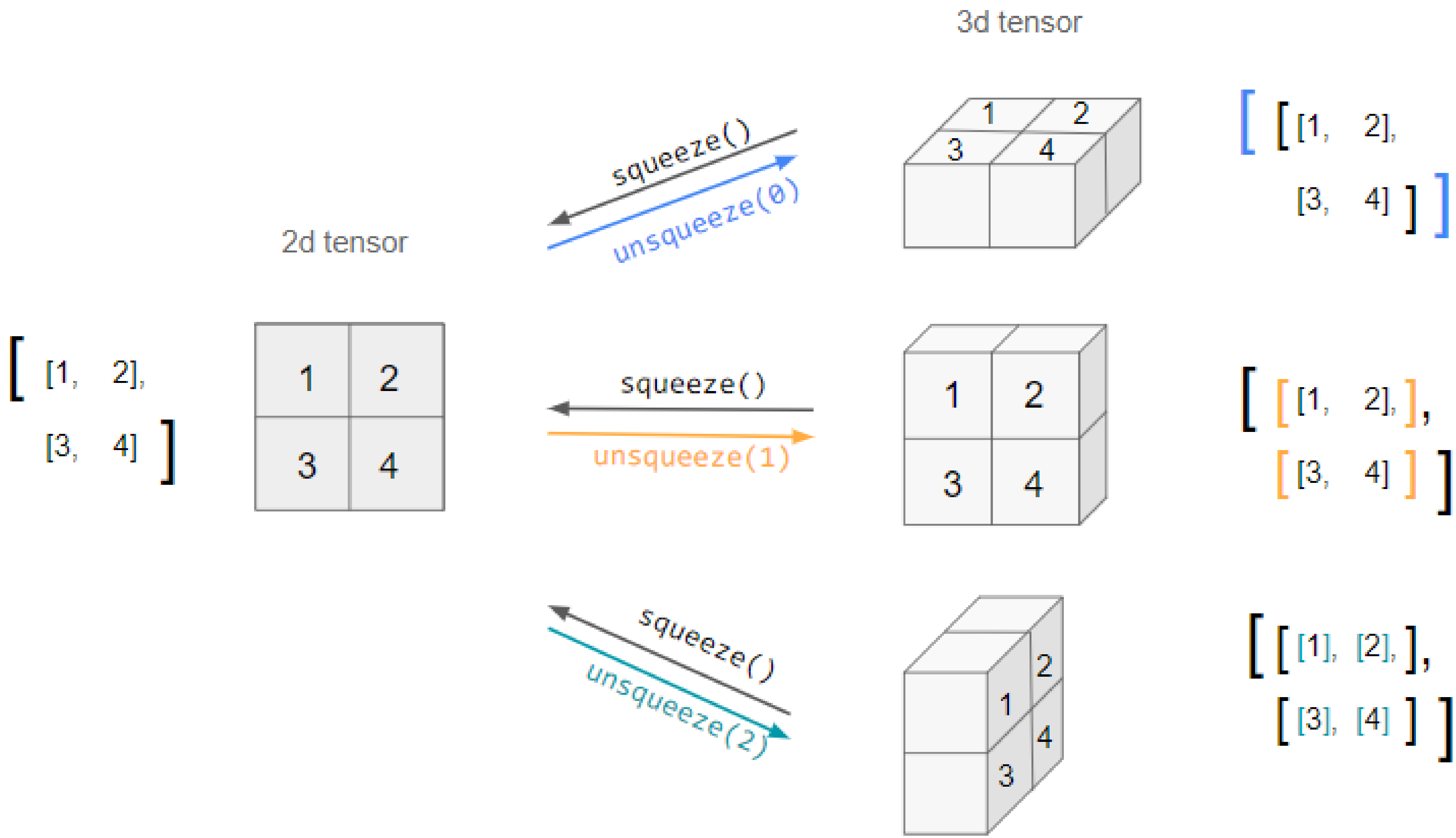
```
torch.Size([1, 3, 2])
```

```
x.unsqueeze(1).shape
```

```
torch.Size([3, 1, 2])
```

```
x.unsqueeze(2).shape
```

```
torch.Size([3, 2, 1])
```



From stackoverflow post by iacob

Exercise 2

Given the following tensors,

```
a = torch.ones(4,3,2)
b = torch.rand(3)
c = torch.rand(5,3)
```

what reshaping is needed to make it possible so that `a * b` and `a * c` can be calculated via broadcasting?

Autograd

Tensor expressions

Gradient tracking can be enabled using the `requires_grad` argument at initialization, alternatively the `requires_grad` flag can be set on the tensor or the `enable_grad()` context manager used (via `with`).

```
x = torch.linspace(0, 2, steps=21, requires_grad=True); x
```

```
tensor([0.00000000, 0.10000000, 0.20000000, 0.30000001, 0.40000001, 0.50000000,
        0.60000002, 0.69999999, 0.80000001, 0.90000004, 1.00000000, 1.09999990,
        1.20000005, 1.29999995, 1.39999998, 1.50000000, 1.60000002, 1.70000005,
        1.79999995, 1.89999998, 2.00000000], requires_grad=True)
```

```
y = 3*x + 2; y
```

```
tensor([2.00000000, 2.29999995, 2.59999990, 2.90000010, 3.20000005, 3.50000000,
        3.80000019, 4.09999990, 4.40000010, 4.69999981, 5.00000000, 5.29999971,
        5.60000038, 5.89999962, 6.19999981, 6.50000000, 6.80000019, 7.10000038,
        7.39999962, 7.69999981, 8.00000000], grad_fn=<AddBackward0>)
```

Computational graph

Basics of the computation graph can be explored via the `next_functions` attribute

```
y.grad_fn
```

```
<AddBackward0 object at 0x29cee8520>
```

```
y.grad_fn.next_functions
```

```
((<MulBackward0 object at 0x29cee8640>, 0), (None, 0))
```

```
y.grad_fn.next_functions[0][0].next_functions
```

```
((<AccumulateGrad object at 0x29cee8490>, 0), (None, 0))
```

```
y.grad_fn.next_functions[0][0].next_functions[0][0].next_functions
```

```
()
```

Autogradient

In order to calculate the gradients we use the `backward()` method on the *output* tensor (must be a scalar), this then makes the grad attribute available for the input (leaf) tensors.

```
out = y.sum()
out.backward()
out
```

```
tensor(105., grad_fn=<SumBackward0>)
```

```
y.grad
```

```
<string>:1: UserWarning: The .grad attribute of a Tensor that is not a leaf Tensor is being accessed. Its .g
   attribute won't be populated during autograd.backward(). If you indeed want the .grad field to be populat
   non-leaf Tensor, use .retain_grad() on the non-leaf Tensor. If you access the non-leaf Tensor by mistake,
   you access the leaf Tensor instead. See github.com/pytorch/pytorch/pull/30531 for more informations. (Tri
   internally at /Users/runner/work/pytorch/pytorch/pytorch/build/aten/src/ATen/core/TensorBody.h:491.)
```

```
x.grad
```

```
tensor([3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3., 3.,
        3., 3., 3.])
```

A bit more complex

```
n = 21
x = torch.linspace(0, 2, steps=n, requires_grad=True)
m = torch.rand(n, requires_grad=True)

y = m*x + 2

y.backward(torch.ones(n))
```

```
x.grad
```

```
tensor([0.23227984, 0.72686875, 0.11874896, 0.39512146, 0.71987736, 0.75950843,
        0.53108865, 0.64494550, 0.72242016, 0.44158769, 0.36338443, 0.88182861,
        0.98741043, 0.73160070, 0.28143251, 0.06507802, 0.00649202, 0.50345892,
        0.30815977, 0.37417805, 0.42968810])
```

```
m.grad
```

```
tensor([0.00000000, 0.10000000, 0.20000000, 0.30000001, 0.40000001, 0.50000000,
        0.60000002, 0.69999999, 0.80000001, 0.90000004, 1.00000000, 1.09999990,
        1.20000005, 1.29999995, 1.39999998, 1.50000000, 1.60000002, 1.70000005,
        1.79999995, 1.89999998, 2.00000000])
```

In context you can interpret `x.grad` and `m.grad` as the gradient of `y` with respect to `x` or `m` respectively.

High-level autograd API

This allows for the automatic calculation and evaluation of the jacobian and hessian for a function defined using tensors.

```
def f(x, y):
  return 3*x + 1 + 2*y**2 + x*y
```

```
for x in [0.,1.]:
  for y in [0.,1.]:
    print("x =",x, "y = ",y)
    inputs = (torch.tensor([x]), torch.tensor([y]))
    print(torch.autograd.functional.jacobian(f, inputs),"\n")
```

```
x = 0.0 y =  0.0
(tensor([[3.]]), tensor([[0.]]))

x = 0.0 y =  1.0
(tensor([[4.]]), tensor([[4.]]))

x = 1.0 y =  0.0
(tensor([[3.]]), tensor([[1.]]))

x = 1.0 y =  1.0
(tensor([[4.]]), tensor([[5.]]))
```

```python
inputs = (torch.tensor([0.]), torch.tensor([0.]))
torch.autograd.functional.hessian(f, inputs)
```

```
((tensor([[0.]]), tensor([[1.]])), (tensor([[1.]]), tensor([[4.]])))
```

```python
inputs = (torch.tensor([1.]), torch.tensor([1.]))
torch.autograd.functional.hessian(f, inputs)
```

```
((tensor([[0.]]), tensor([[1.]])), (tensor([[1.]]), tensor([[4.]])))
```

Demo 1 - Linear Regression w/ PyTorch

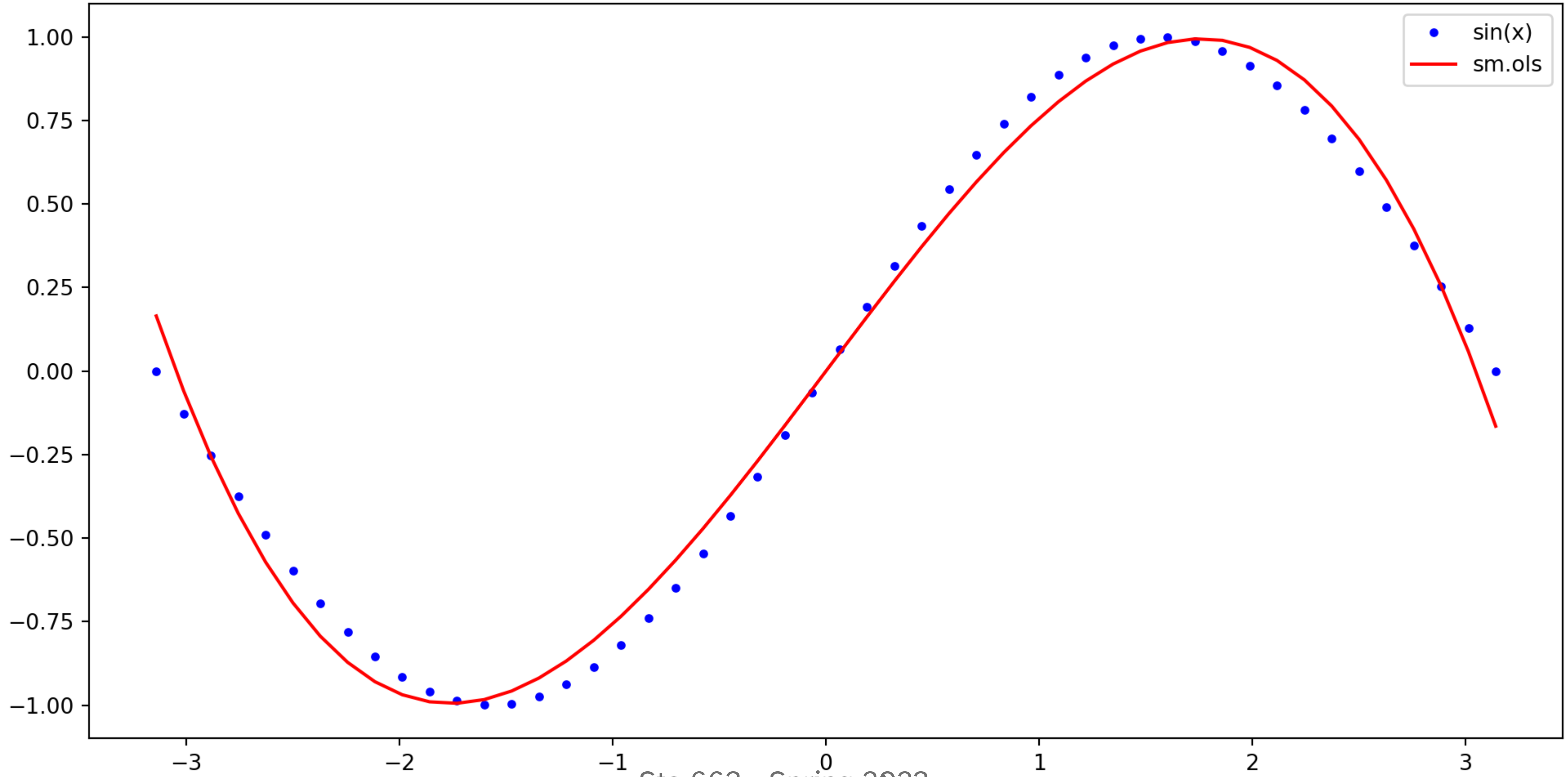
A basic model

```python
x = np.linspace(-math.pi, math.pi, 50)
y = np.sin(x)

lm = smf.ols(
  "y~x+I(x**2)+I(x**3)",
  data=pd.DataFrame({"x": x, "y": y})
).fit()

print(lm.summary())
```

```
                            OLS Regression Results
=====================================================
Dep. Variable:                      y   R-squared:
Model:                            OLS   Adj. R-square
Method:                 Least Squares   F-statistic:
Date:                Fri, 07 Apr 2023   Prob (F-stat
Time:                        12:43:41   Log-Likelihoo
No. Observations:                  50   AIC:
Df Residuals:                      46   BIC:
Df Model:                           3
Covariance Type:            nonrobust
=====================================================
                 coef    std err          t      P>|t
-----------------------------------------------------
Intercept   3.161e-16      0.016      2e-14      1.0
x              0.8476      0.014     59.444      0.0
I(x ** 2)  -4.949e-17      0.003  -1.44e-14      1.0
I(x ** 3)     -0.0912      0.002    -42.977      0.0
=====================================================
Omnibus:                        3.322   Durbin-Watso
Prob(Omnibus):                  0.190   Jarque-Bera
Skew:                           0.000   Prob(JB):
Kurtosis:                       2.111   Cond. No.
```

Predictions

```
plt.figure(figsize=(10,5), layout="constrained")
plt.plot(x, y, ".b", label="sin(x)")
plt.plot(x, lm.predict(), "-r", label="sm.ols")
plt.legend()
plt.show()
```



Making tensors

```python
yt = torch.tensor(y)
Xt = torch.tensor(lm.model.exog)
bt = torch.randn((Xt.shape[1], 1), dtype=torch.float64, requires_grad=True)
```

```python
yt
```

```
tensor([-1.22464680e-16, -1.27877162e-01, -2.5365458
        -4.90717552e-01, -5.98110530e-01, -6.9568255
        -8.55142763e-01, -9.14412623e-01, -9.5866785
        -9.99486216e-01, -9.95379113e-01, -9.7492791
        -8.86599306e-01, -8.20172255e-01, -7.4027799
        -5.45534901e-01, -4.33883739e-01, -3.1510821
        -6.40702200e-02,  6.40702200e-02,  1.9115862
         4.33883739e-01,  5.45534901e-01,  6.4822839
```

```python
Xt
```

```
tensor([[ 1.00000000e+00, -3.14159265e+00,  9.869604
        [ 1.00000000e+00, -3.01336438e+00,  9.080364
        [ 1.00000000e+00, -2.88513611e+00,  8.324010
        [ 1.00000000e+00, -2.75690784e+00,  7.600540
        [ 1.00000000e+00, -2.62867957e+00,  6.909956
        [ 1.00000000e+00, -2.50045130e+00,  6.252256
        [ 1.00000000e+00, -2.37222302e+00,  5.627442
        [ 1.00000000e+00, -2.24399475e+00,  5.035512
```

```python
yt_pred = (Xt @ bt).squeeze()
```

```python
loss = (yt_pred - yt).pow(2).sum()
loss.item()
```

```
2119.2777040165224
```

Gradient descent

Going back to our discussion of optimization and gradient descent awhile back - we can update our guess for `b` / `bt` by moving in the direction of the negative gradient. The step size is refered to as the *learning rate* which we will pick a relatively small value for.

```
learning_rate = 1e-6

loss.backward() # Compute the backward pass

with torch.no_grad():
  bt -= learning_rate * bt.grad # Make the step

  bt.grad = None # Reset the gradients
```

```
yt_pred = (Xt @ bt).squeeze()
loss = (yt_pred - yt).pow(2).sum()
loss.item()
```

```
2069.4881821807053
```

Putting it together

```
yt = torch.tensor(y).unsqueeze(1)
Xt = torch.tensor(lm.model.exog)
bt = torch.randn((Xt.shape[1], 1), dtype=torch.float64, requires_grad=True)

learning_rate = 1e-5
for i in range(5000):

  yt_pred = Xt @ bt

  loss = (yt_pred - yt).pow(2).sum()
  if i % 500 == 0:
    print(f"Step: {i},\tloss: {loss.item()}")

  loss.backward()

  with torch.no_grad():
    bt -= learning_rate * bt.grad
    bt.grad = None
```

Putting it together

```
Step: 0,    loss: 70161.1580804254
Step: 500,  loss: 14.791178300540242
Step: 1000, loss: 8.825181658035252
Step: 1500, loss: 5.311942717260375
Step: 2000, loss: 3.2416251317783518
Step: 2500, loss: 2.020671792951764
Step: 3000, loss: 1.3000220383569292
Step: 3500, loss: 0.8742816442183534
Step: 4000, loss: 0.6225166364100523
Step: 4500, loss: 0.473473387453477
```

```
print(bt)
```

```
tensor([[ 0.03143311],
        [ 0.78484316],
        [-0.00520945],
        [-0.08260584]], dtype=torch.float64, requires_grad=True)
```

Comparing results

```
1  bt
```

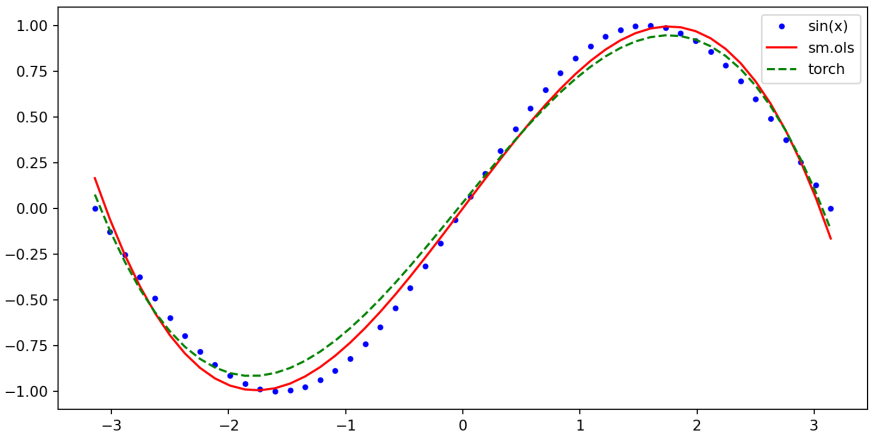
```
tensor([[ 0.03143311],
        [ 0.78484316],
        [-0.00520945],
        [-0.08260584]], dtype=torch.float64, require
```

```
1  lm.params
```

```
Intercept     3.160866e-16
x             8.476289e-01
I(x ** 2)    -4.949020e-17
I(x ** 3)    -9.120167e-02
dtype: float64
```


```
1  bt
```

```
tensor([[ 0.03143311],
        [ 0.78484316],
        [-0.00520945],
        [-0.08260584]], dtype=torch.float64, require
```

Demo 2 - Using a torch model

A sample model

```
class Model(torch.nn.Module):
    def __init__(self, beta):
        super().__init__()
        beta.requires_grad = True
        self.beta = torch.nn.Parameter(beta)

    def forward(self, X):
        return X @ self.beta

def training_loop(model, X, y, optimizer, n=1000):
    losses = []
    for i in range(n):
        y_pred = model(X)

        loss = (y_pred.squeeze() - y.squeeze()).pow(2).sum()
        loss.backward()

        optimizer.step()
        optimizer.zero_grad()

        losses.append(loss.item())

    return losses
```

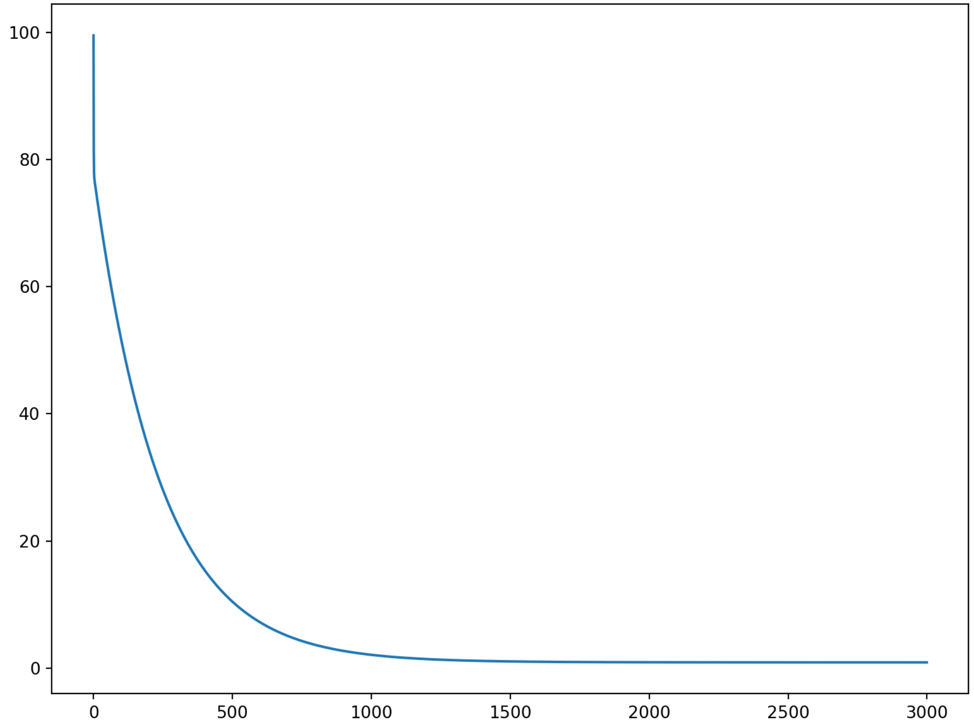
Fitting

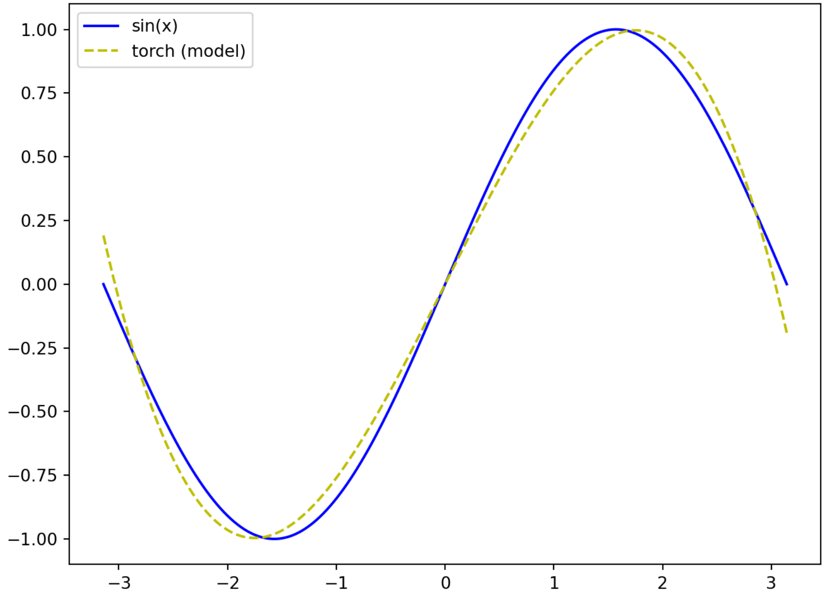
```
x = torch.linspace(-math.pi, math.pi, 200)
y = torch.sin(x)

X = torch.vstack((
  torch.ones_like(x),
  x,
  x**2,
  x**3
)).T

m = Model(beta = torch.zeros(4))
opt = torch.optim.SGD(m.parameters(), lr=1e-5)

losses = training_loop(m, X, y, opt, n=3000)
```

Results

```
m.beta
```

```
Parameter containing:
tensor([ 2.66870664e-10,  8.52953434e-01,  6.79866718e-11, -9.25917700e-02],
       requires_grad=True)
```



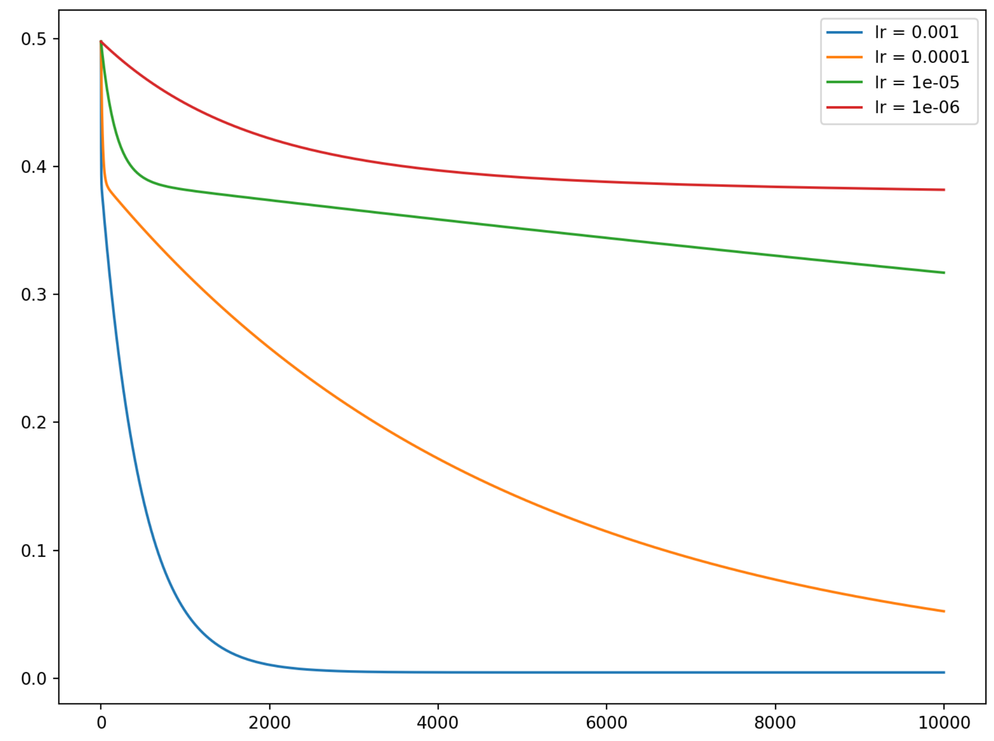


An all-in-one model

```
class Model(torch.nn.Module):
    def __init__(self, X, y, beta=None):
        super().__init__()
        self.X = X
        self.y = y
        if beta is None:
          beta = torch.zeros(X.shape[1])
        beta.requires_grad = True
        self.beta = torch.nn.Parameter(beta)

    def forward(self, X):
        return X @ self.beta

    def fit(self, opt, n=1000, loss_fn = torch.nn.MSELoss()):
      losses = []
      for i in range(n):
          loss = loss_fn(self(self.X).squeeze(), self.y.squeeze())
          loss.backward()
          opt.step()
          opt.zero_grad()
          losses.append(loss.item())

      return losses
```

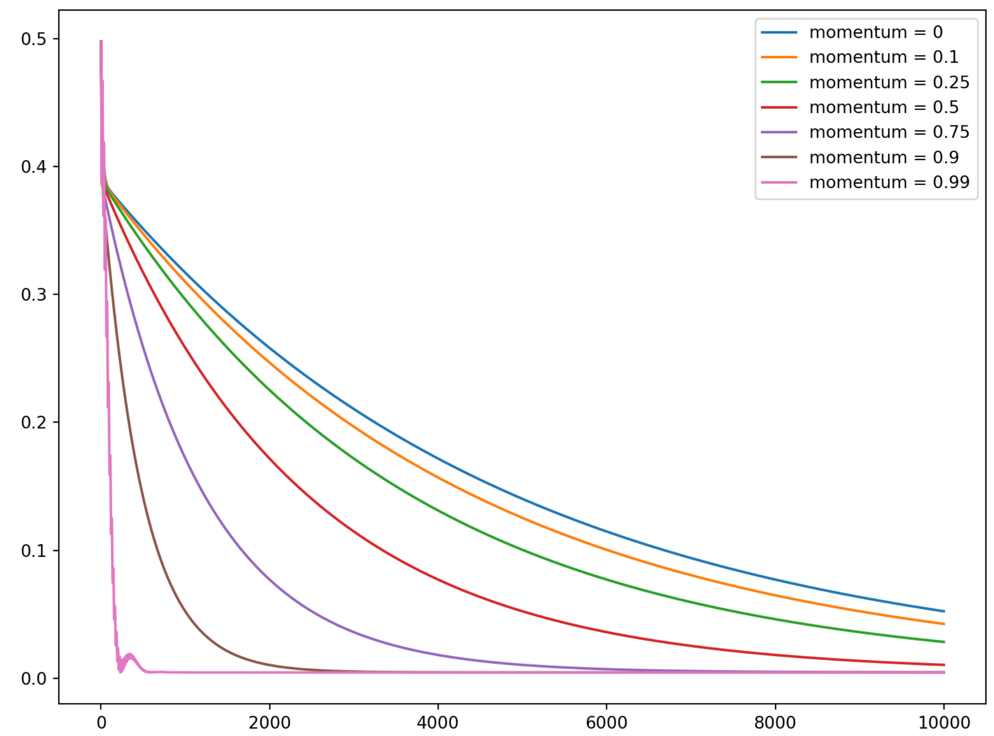
Learning rate and convergence

```
plt.figure(figsize=(8,6), layout="constrained")

for lr in [1e-3, 1e-4, 1e-5, 1e-6]:
  m = Model(X, y)
  opt = torch.optim.SGD(m.parameters(), lr=lr)
  losses = m.fit(opt, n=10000)

  plt.plot(losses, label=f"lr = {lr}")

plt.legend()
plt.show()
```



Momentum and convergence

```
plt.figure(figsize=(8,6), layout="constrained")

for mt in [0, 0.1, 0.25, 0.5, 0.75, 0.9, 0.99]:
  m = Model(X, y)
  opt = torch.optim.SGD(
    m.parameters(),
    lr = 1e-4,
    momentum = mt
  )
  losses = m.fit(opt, n=10000)

  plt.plot(losses, label=f"momentum = {mt}")

plt.legend()
plt.show()
```



Optimizers and convergence

```python
plt.figure(figsize=(8,6), layout="constrained")

opts = (torch.optim.SGD,
        torch.optim.Adam,
        torch.optim.Adagrad)

for opt_fn in opts:
  m = Model(X, y)
  opt = opt_fn(m.parameters(), lr=1e-4)
  losses = m.fit(opt, n=10000)

  plt.plot(losses, label=f"opt = {opt_fn}")

plt.legend()
plt.show()
```