

# PyMC + ArviZ

## Lecture 19

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# pymc

PyMC is a probabilistic programming library for Python that allows users to build Bayesian models with a simple Python API and fit them using Markov chain Monte Carlo (MCMC) methods.

- **Modern** - Includes state-of-the-art inference algorithms, including MCMC (NUTS) and variational inference (ADVI).
- **User friendly** - Write your models using friendly Python syntax. Learn Bayesian modeling from the many example notebooks.
- **Fast** - Uses Aesara as its computational backend to compile to C and JAX, run your models on the GPU, and benefit from complex graph-optimizations.
- **Batteries included** - Includes probability distributions, Gaussian processes, ABC, SMC and much more. It integrates nicely with ArviZ for visualizations and diagnostics, as well as Bambi for high-level mixed-effect models.
- **Community focused** - Ask questions on discourse, join MeetUp events, follow us on Twitter, and start contributing.

```
1 import pymc as pm
```

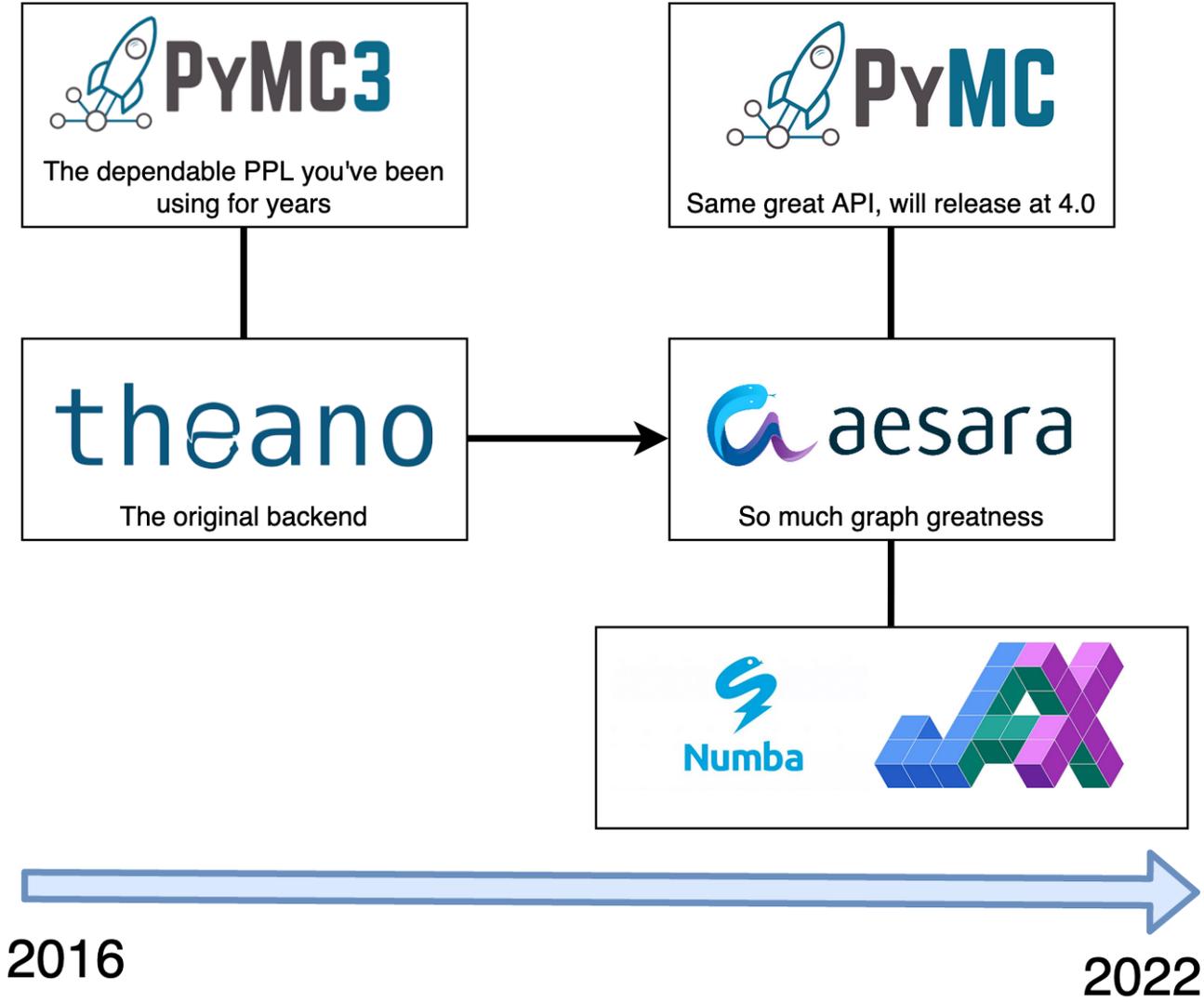
# ArviZ

ArviZ is a Python package for exploratory analysis of Bayesian models. Includes functions for posterior analysis, data storage, sample diagnostics, model checking, and comparison.

- **Interoperability** - Integrates with all major probabilistic programming libraries: PyMC, CmdStanPy, PyStan, Pyro, NumPyro, and emcee.
- **Large Suite of Visualizations** - Provides over 25 plotting functions for all parts of Bayesian workflow: visualizing distributions, diagnostics, and model checking. See the gallery for examples.
- **State of the Art Diagnostics** - Latest published diagnostics and statistics are implemented, tested and distributed with ArviZ.
- **Flexible Model Comparison** - Includes functions for comparing models with information criteria, and cross validation (both approximate and brute force).
- **Built for Collaboration** - Designed for flexible cross-language serialization using netCDF or Zarr formats. ArviZ also has a Julia version that uses the same data schema.
- **Labeled Data** - Builds on top of xarray to work with labeled dimensions and coordinates.

```
1 import arviz as az
```

# Some history



# Model basics

All models are derived from the `Model()` class, unlike what we have seen previously PyMC makes heavy use of Python's context manager using the `with` statement to add model components to a model.

```
1 with pm.Model() as norm:  
2     x = pm.Normal("x", mu=0, sigma=1)
```

```
1 x = pm.Normal("x", mu=0, sigma=1)
```

Error: `TypeError: No model on context stack, which is needed to instantiate distributions. Add variable inside`

Note that `with` blocks do not have their own scope - so variables defined inside are added to the parent scope (be careful about overwriting other variables).

```
1 x
```

x

```
1 type(x)
```

```
<class 'pytensor.tensor.var.TensorVariable'>
```

# Random Variables

`pm.Normal()` is an example of a PyMC distribution, which are used to construct models, these are implemented using the `TensorVariable` class which is used for all of the builtin distributions (and can be used to create custom distributions). Generally you will not be interacting with these objects directly, but with that said some useful methods and attributes:

```
1 type(norm.x)
```

```
<class 'pytensor.tensor.var.TensorVariable'>
```

```
1 norm.x.owner.op
```

```
<pytensor.tensor.random.basic.NormalRV object at 0x17280e980>
```

```
1 pm.draw(norm.x)
```

```
array(0.16579)
```

# Standalone RVs

If you really want to construct a `TensorVariable` outside of a model this can be done via the `dist` method for each distribution.

```
1 z = pm.Normal.dist(mu=1, sigma=2, shape=[2,3])
2 z
```

```
normal_rv{0, (0, 0), floatX, False}.out
```

```
1 pm.draw(z)
```

```
array([[ 1.83488,  4.45201, -1.60857],
       [-0.29248,  1.7887 ,  1.81323]])
```

# Modifying models

Because of this construction it is possible to add additional components to an existing (named) model via subsequent `with` statements (only the first needs `pm.Model()`)

```
1 with norm:  
2     y = pm.Normal("y", mu=x, sigma=1, shape=3)
```

```
1 norm.basic_RVs
```

[x, y]

# Variable hierarchy

Note that we defined  $y|x \sim (x, 1)$ , so what is happening when we use `pm.draw(norm.y)`?

```
1 pm.draw(norm.y)
```

```
array([-1.49714, -1.75064, -0.81631])
```

```
1 obs = pm.draw(norm.y, draws=1000); obs
```

```
array([[ 2.2078 ,  2.29494,  1.4284 ],
       [-0.93994, -0.55348,  0.15047],
       [ 0.54024,  1.27989,  0.67047],
       ...,
       [ 1.6475 ,  0.83945,  0.71665],
       [-0.18381, -0.30989, -0.36771],
       [-0.52248, -0.31888, -0.42917]])
```

```
1 np.mean(obs)
```

```
0.06400085943586256
```

```
1 np.var(obs)
```

```
1.9553522392586231
```

```
1 np.std(obs)
```

```
1.3983391002395031
```

Each time we ask for a draw from  $y$ , PyMC is first drawing from  $x$  for us.

# Beta-Binomial model

We will now build a basic model where we know what the solution should look like and compare the results.

```
1 with pm.Model() as beta_binom:
2     p = pm.Beta("p", alpha=10, beta=10)
3     x = pm.Binomial("x", n=20, p=p, observed=5)
```

In order to sample from the posterior we add a call to `sample()` within the model context.

```
1 with beta_binom:
2     trace = pm.sample(random_seed=1234, progressbar=False)
```

Auto-assigning NUTS sampler...

Initializing NUTS using jitter+adapt\_diag...

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [p]

Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws total) took 0 seconds.

# pm.sample() results

```
1 print(trace)
```

Inference data with groups:

```
> posterior  
> sample_stats  
> observed_data
```

```
1 print(type(trace))
```

```
<class 'arviz.data.inference_data.InferenceData'>
```

# Xarray - N-D labeled arrays and datasets in Python

Xarray (formerly xray) is an open source project and Python package that makes working with labelled multi-dimensional arrays simple, efficient, and fun!

Xarray introduces labels in the form of dimensions, coordinates and attributes on top of raw NumPy-like arrays, which allows for a more intuitive, more concise, and less error-prone developer experience. The package includes a large and growing library of domain-agnostic functions for advanced analytics and visualization with these data structures.

Xarray is inspired by and borrows heavily from pandas, the popular data analysis package focused on labelled tabular data. It is particularly tailored to working with netCDF files, which were the source of xarray's data model, and integrates tightly with dask for parallel computing.

# Digging into trace

```
1 print(trace.posterior)
```

```
<xarray.Dataset>
```

```
Dimensions: (chain: 4, draw: 1000)
```

```
Coordinates:
```

```
* chain      (chain) int64 0 1 2 3
```

```
* draw       (draw) int64 0 1 2 3 4 5 6 7 8 ... 992 993 994 995 996 997 998 999
```

```
Data variables:
```

```
  p          (chain, draw) float64 0.5068 0.4518 0.3853 ... 0.166 0.3242 0.3242
```

```
Attributes:
```

```
  created_at:          2023-03-24T16:28:00.773567
```

```
  arviz_version:       0.15.1
```

```
  inference_library:   pymc
```

```
  inference_library_version: 5.1.2
```

```
  sampling_time:       0.32303428649902344
```

```
  tuning_steps:        1000
```

```
1 print(trace.posterior["p"].shape)
```

```
(4, 1000)
```

```
1 print(trace.sel(chain=0).posterior["p"].shape)
```

```
(1000,)
```

```
1 print(trace.sel(draw=slice(500, None, 10)).posterior["p"].shape)
```

```
(4, 50)
```

# As DataFrame

Posterior values, or subsets, can be converted to DataFrames via the `to_dataframe()` method

```
1 trace.posterior.to_dataframe()
```

```
          p
chain draw
0      0  0.506797
      1  0.451803
      2  0.385329
      3  0.507061
      4  0.374281
...
3     995 0.284547
      996 0.202096
      997 0.165955
      998 0.324233
      999 0.324233
```

```
[4000 rows x 1 columns]
```

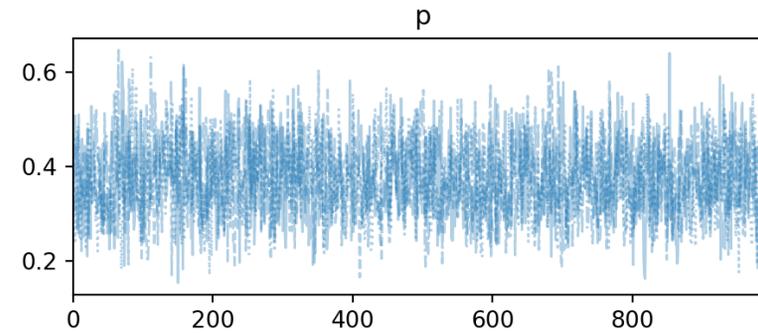
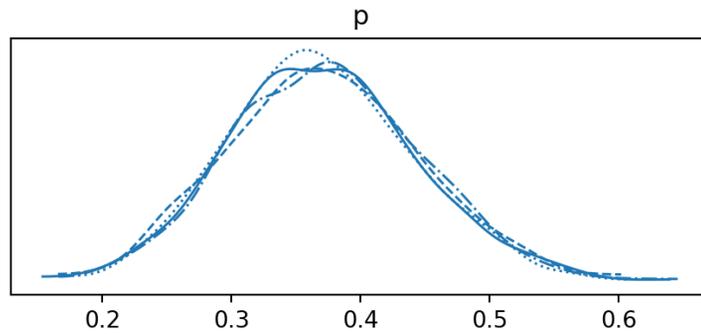
```
1 trace.posterior["p"][0,:].to_dataframe()
```

```
      chain      p
draw
0         0  0.506797
1         0  0.451803
2         0  0.385329
3         0  0.507061
4         0  0.374281
...
995       0  0.356664
996       0  0.356664
997       0  0.397380
998       0  0.401865
999       0  0.401865
```

```
[1000 rows x 2 columns]
```

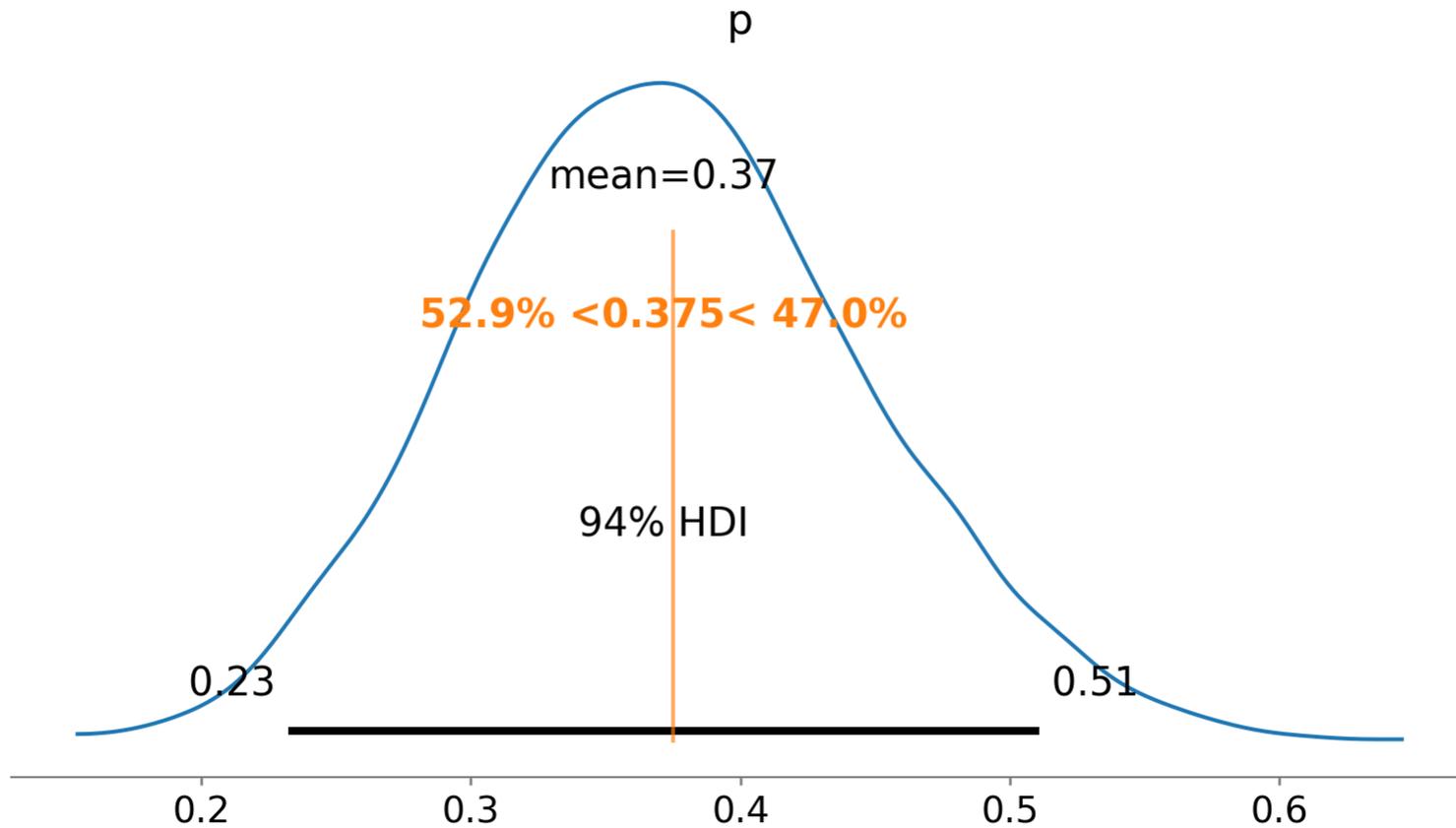
# Traceplot

```
1 ax = az.plot_trace(trace)
2 plt.show()
```



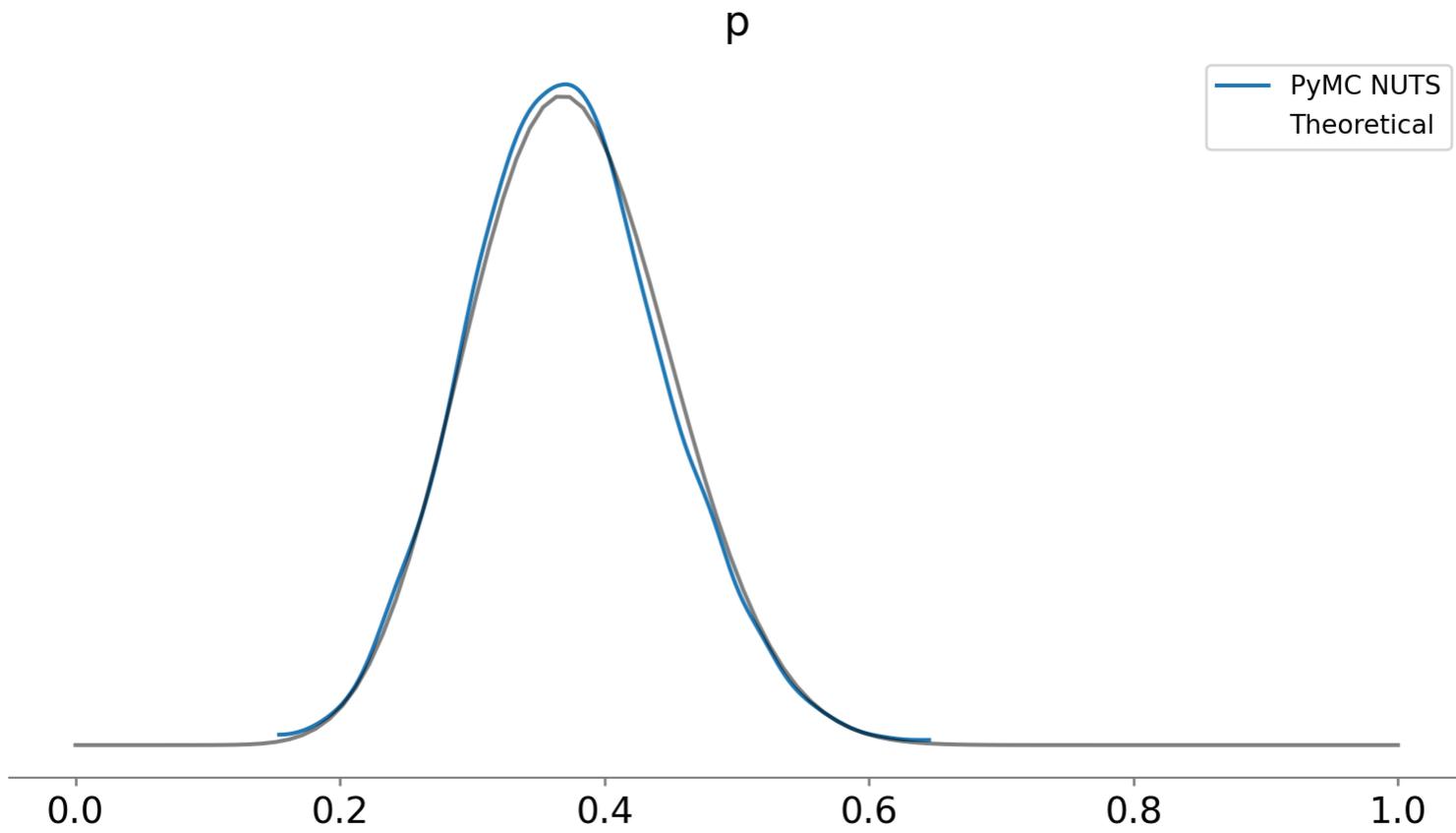
# Posterior plot

```
1 ax = az.plot_posterior(trace, ref_val=[15/40])  
2 plt.show()
```



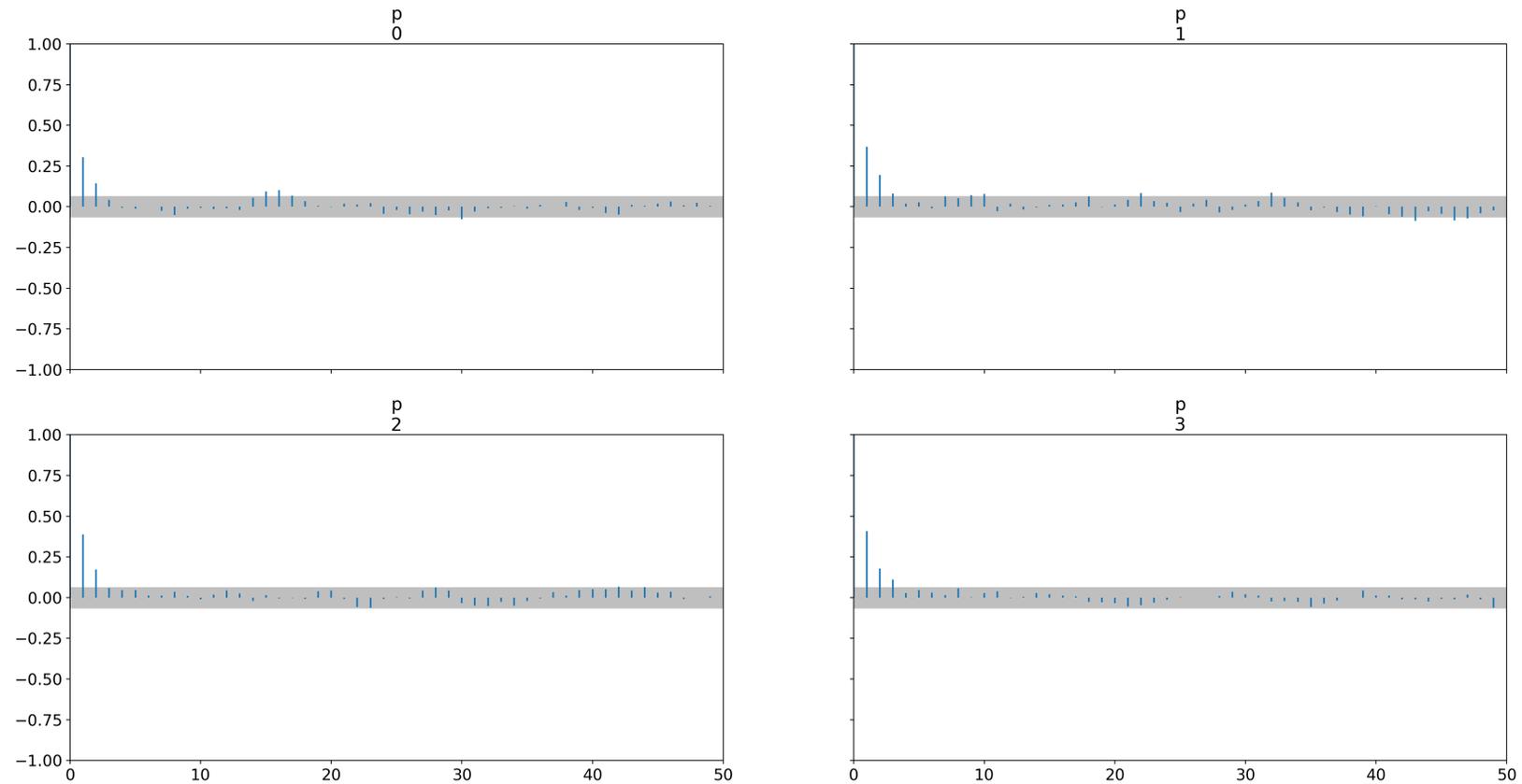
# PyMC vs Theoretical

```
1 p = np.linspace(0, 1, 100)
2 post_beta = scipy.stats.beta.pdf(p,15,25)
3 ax = az.plot_posterior(trace, hdi_prob="hide", point_estimate=None)
4 plt.plot(p, post_beta, "-k", alpha=0.5, label="Theoretical")
5 plt.legend(['PyMC NUTS', 'Theoretical'])
6 plt.show()
```



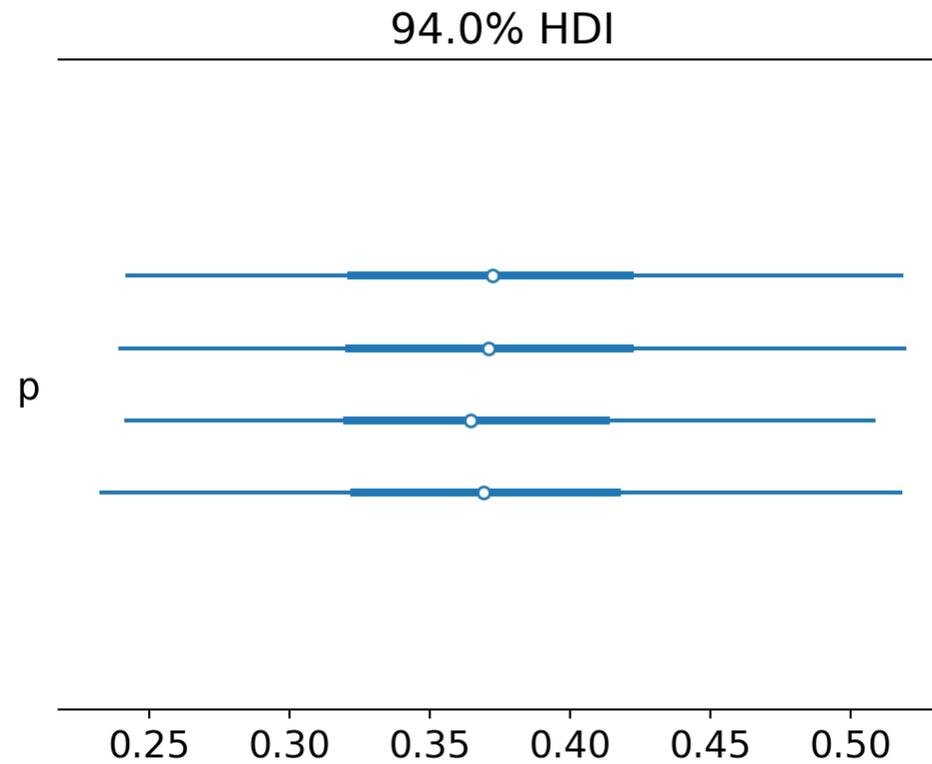
# Autocorrelation plots

```
1 ax = az.plot_autocorr(trace, grid=(2,2), max_lag=50)
2 plt.show()
```



# Forest plots

```
1 ax = az.plot_forest(trace)
2 plt.show()
```



# Other useful diagnostics

Standard MCMC diagnostic statistics are available via `summary()` from ArviZ

```
1 az.summary(trace)
```

```
      mean      sd  hdi_3%  hdi_97%  mcse_mean  mcse_sd  ess_bulk  ess_tail  r_hat
p  0.372  0.074   0.232   0.511     0.002    0.001   1615.0   2222.0    1.0
```

individual methods are available for each statistics,

```
1 print(az.ess(trace, method="bulk"))
```

```
<xarray.Dataset>
Dimensions:  ()
Data variables:
      p          float64 1.615e+03
```

```
1 print(az.ess(trace, method="tail"))
```

```
<xarray.Dataset>
Dimensions:  ()
Data variables:
      p          float64 2.222e+03
```

```
1 print(az.rhat(trace))
```

```
<xarray.Dataset>
Dimensions:  ()
Data variables:
      p          float64 1.002
```

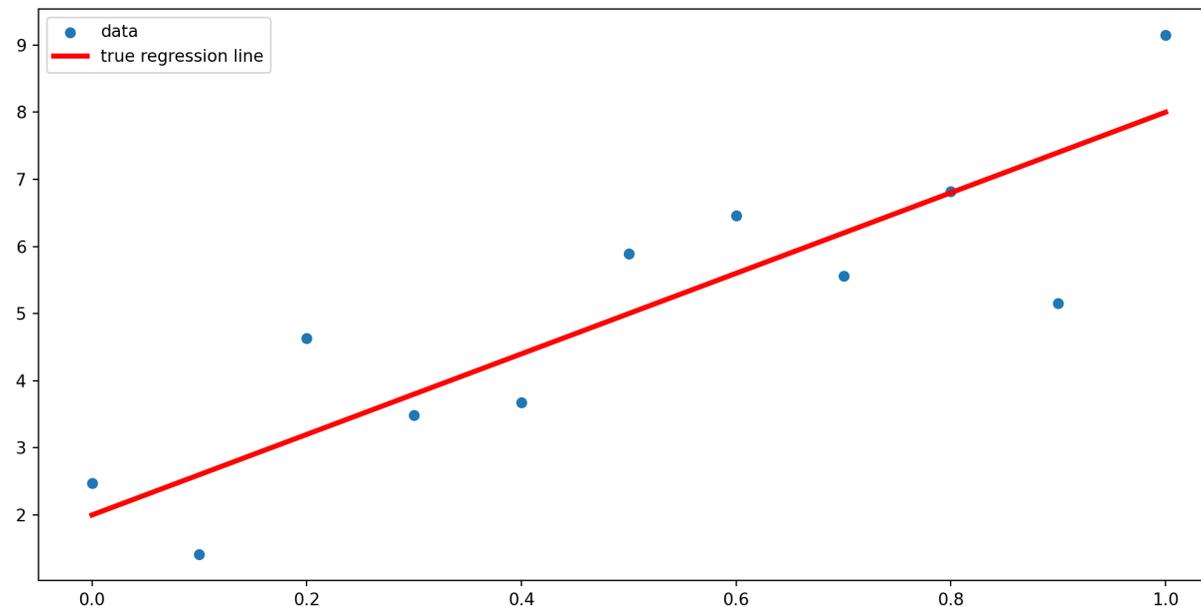
```
1 print(az.mcse(trace))
```

```
<xarray.Dataset>
Dimensions:  ()
Data variables:
      p          float64 0.001844
```

# Demo 1 - Linear regression

Given the below data, we want to fit a linear regression model to the following synthetic data,

```
1 np.random.seed(1234)
2 n = 11
3 m = 6
4 b = 2
5 x = np.linspace(0, 1, n)
6 y = m*x + b + np.random.randn(n)
```



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# Model

```
1 with pm.Model() as lm:
2     m = pm.Normal('m', mu=0, sigma=50)
3     b = pm.Normal('b', mu=0, sigma=50)
4     sigma = pm.HalfNormal('sigma', sigma=5)
5
6     pm.Normal('y', mu=m*x + b, sigma=sigma, observed=y)
7
8     trace = pm.sample(progressbar=False, random_seed=1234)
```

y

Auto-assigning NUTS sampler...

Initializing NUTS using jitter+adapt\_diag...

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [m, b, sigma]

Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws t

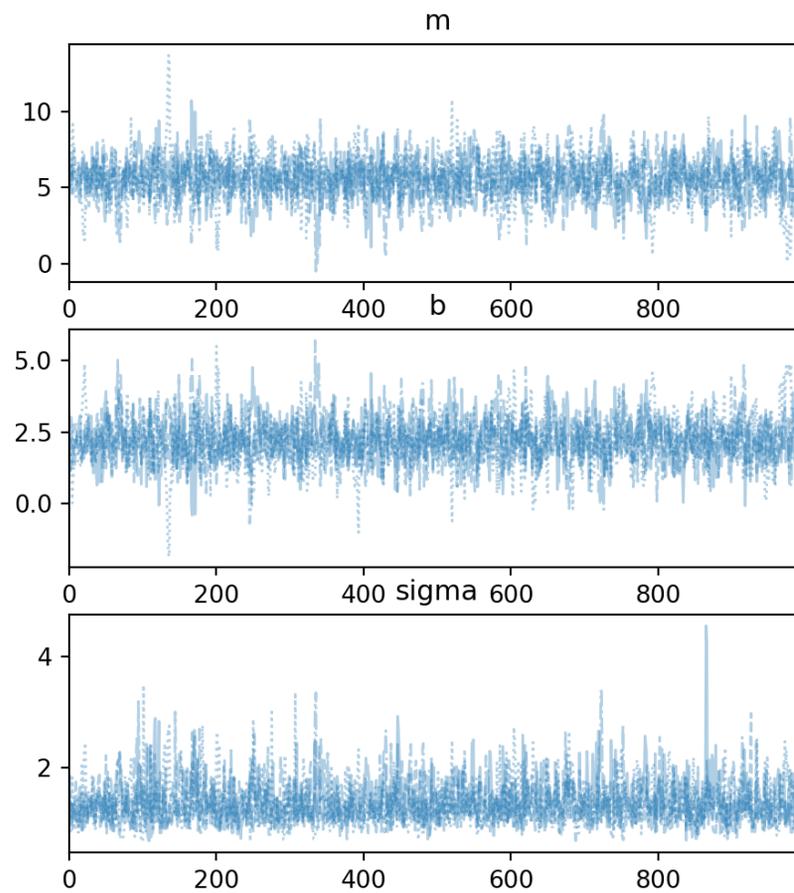
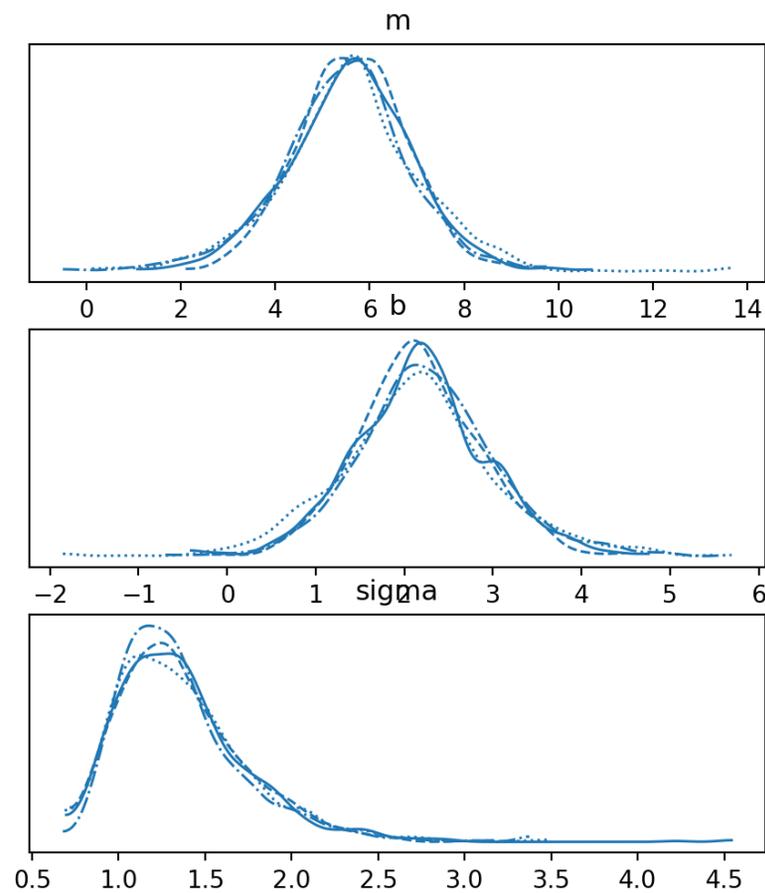
# Posterior summary

```
1 az.summary(trace)
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
m	5.573	1.344	2.969	8.042	0.040	0.028	1182.0	1229.0	1.0
b	2.189	0.791	0.659	3.624	0.024	0.018	1106.0	1153.0	1.0
sigma	1.366	0.378	0.788	2.075	0.010	0.007	1452.0	1722.0	1.0

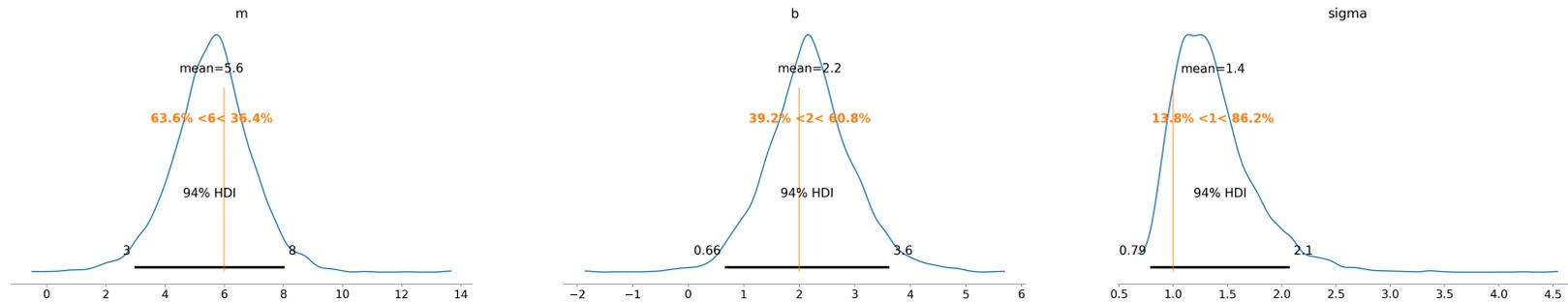
# Trace plots

```
1 ax = az.plot_trace(trace)
2 plt.show()
```



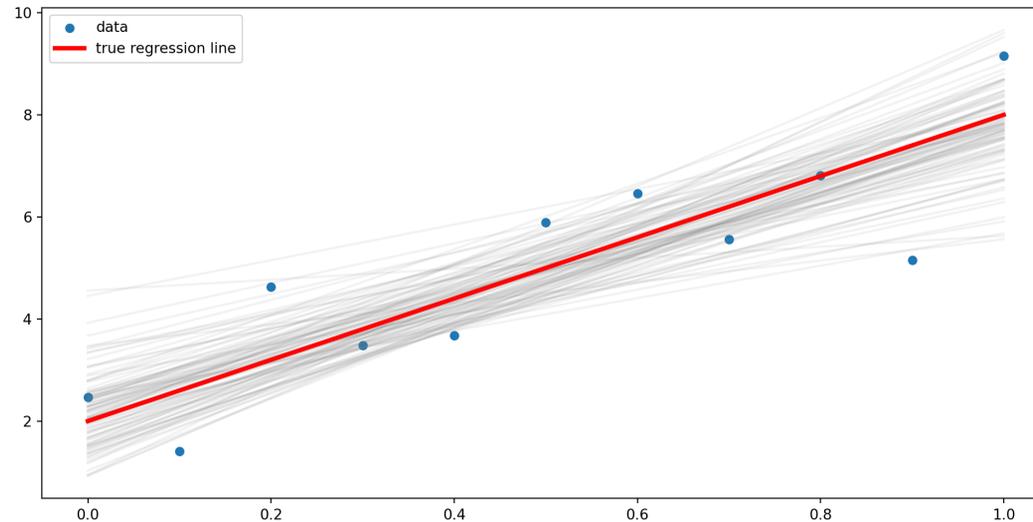
# Posterior plots

```
1 ax = az.plot_posterior(trace, ref_val=[6,2,1], grid=(1,3))  
2 plt.show()
```



# Regression line posterior draws

```
1 plt.scatter(x, y, s=30, label='data')
2
3 post_m = trace.posterior['m'].sel(chain=0, draw=slice(0, None, 10))
4 post_b = trace.posterior['b'].sel(chain=0, draw=slice(0, None, 10))
5
6 plt.figure(layout="constrained")
7 plt.scatter(x, y, s=30, label='data')
8 for m, b in zip(post_m.values, post_b.values):
9     plt.plot(x, m*x + b, c='gray', alpha=0.1)
10 plt.plot(x, 6*x + 2, label='true regression line', lw=3., c='red')
11 plt.legend(loc='best')
12 plt.show()
```



# Posterior predictive draws

Draws for observed variables can also be generated (posterior predictive draws) via the `sample_posterior_predictive()` method.

```
1 with lm:  
2     pp = pm.sample_posterior_predictive(trace, progressbar=False)
```

Sampling: [y]

```
1 pp
```

arviz.InferenceData

- ▶ posterior\_predictive
- ▶ observed\_data

```
1 pp.posterior_predictive
```

<xarray.Dataset>

Dimensions: (chain: 4, draw: 1000, y\_dim\_2: 11)

Coordinates:

\* chain (chain) int64 0 1 2 3

\* draw (draw) int64 0 1 2 3 4 5 6 7 8 ... 992 993 994 995 996 997 998 999

\* y\_dim\_2 (y\_dim\_2) int64 0 1 2 3 4 5 6 7 8 9 10

Data variables:

y (chain, draw, y\_dim\_2) float64 2.857 2.19 4.669 ... 7.195 5.406

Attributes:

created\_at:

2023-03-24T16:28:16.241911

arviz\_version: 0.15.1  
inference\_library: pymc  
inference\_library\_version: 5.1.2

xarray.Dataset

---

► Dimensions:

(chain: 4, draw: 1000, y\_dim\_2: 11)

▼ Coordinates:

**chain**  
(chain) int64  
0 1 2 3



**draw**  
(draw) int64  
0 1 2 3 4 5 ... 995 996 997 998 999



**y\_dim\_2**  
(y\_dim\_2) int64  
0 1 2 3 4 5 6 7 8 9 10



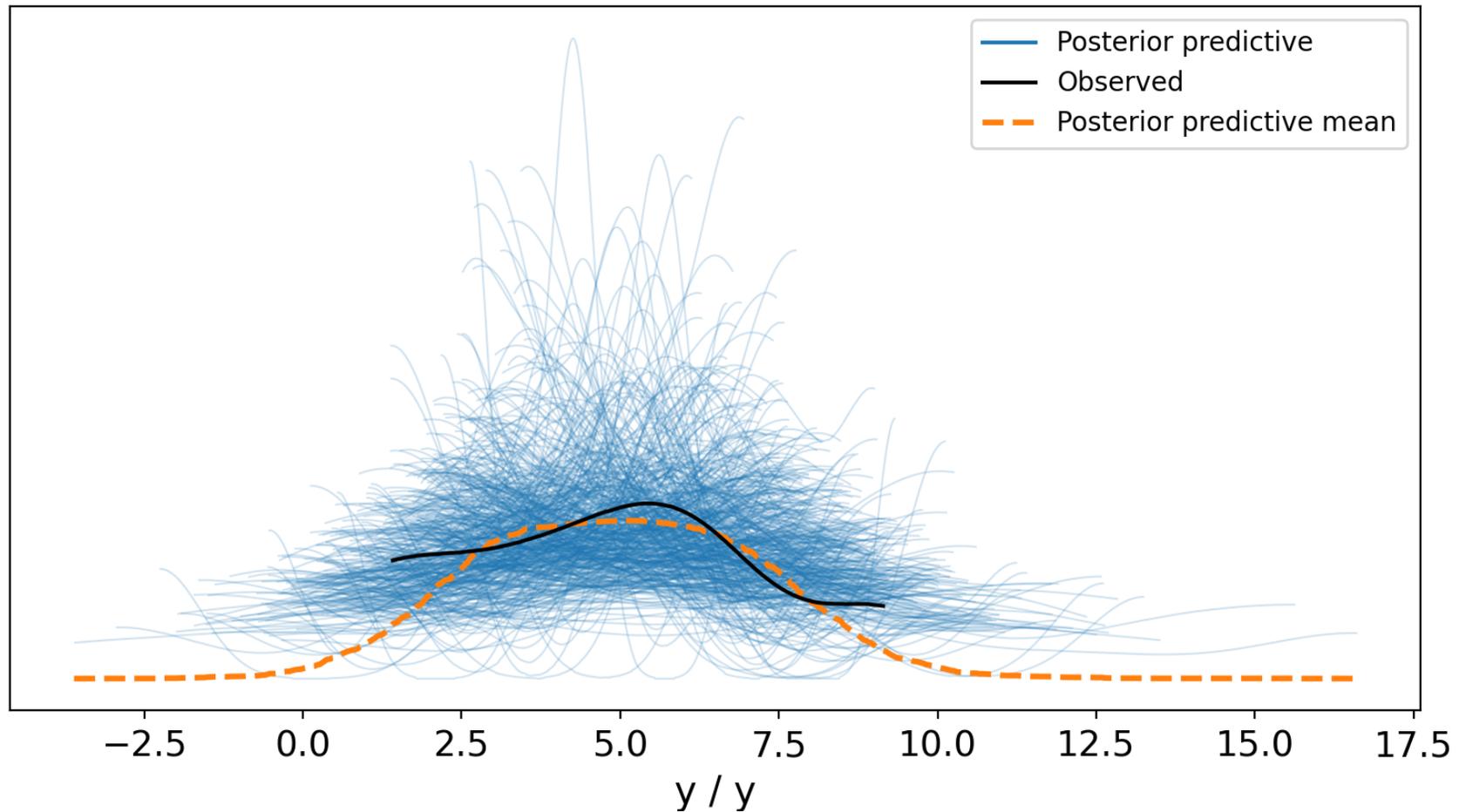
▼ Data variables:

**y**  
(chain, draw, y\_dim\_2) float64  
2.857 2.19 4.669 ... 7.195 5.406



# Plotting the posterior predictive distribution

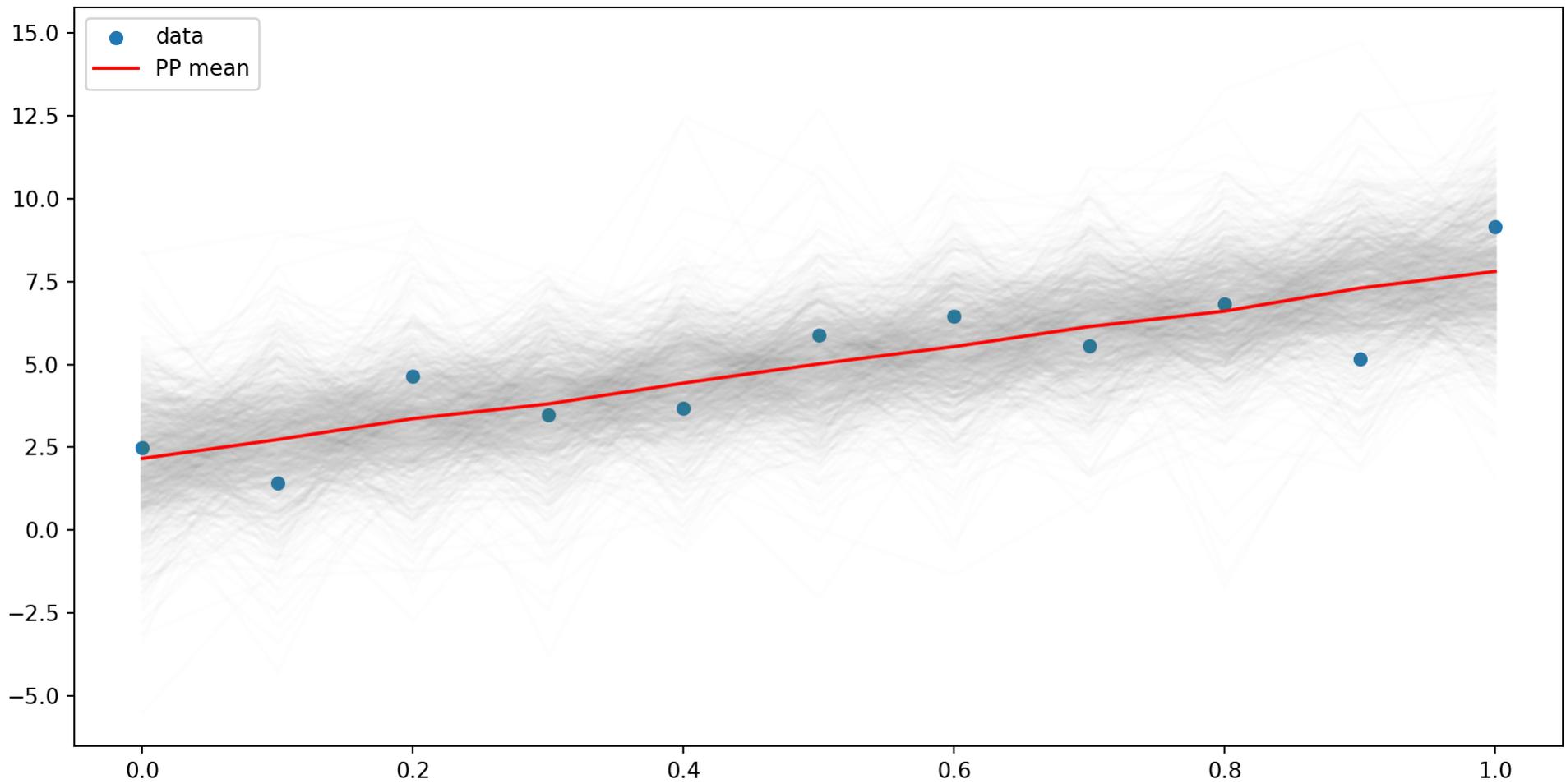
```
1 az.plot_ppc(pp, num_pp_samples=500)
2 plt.show()
```



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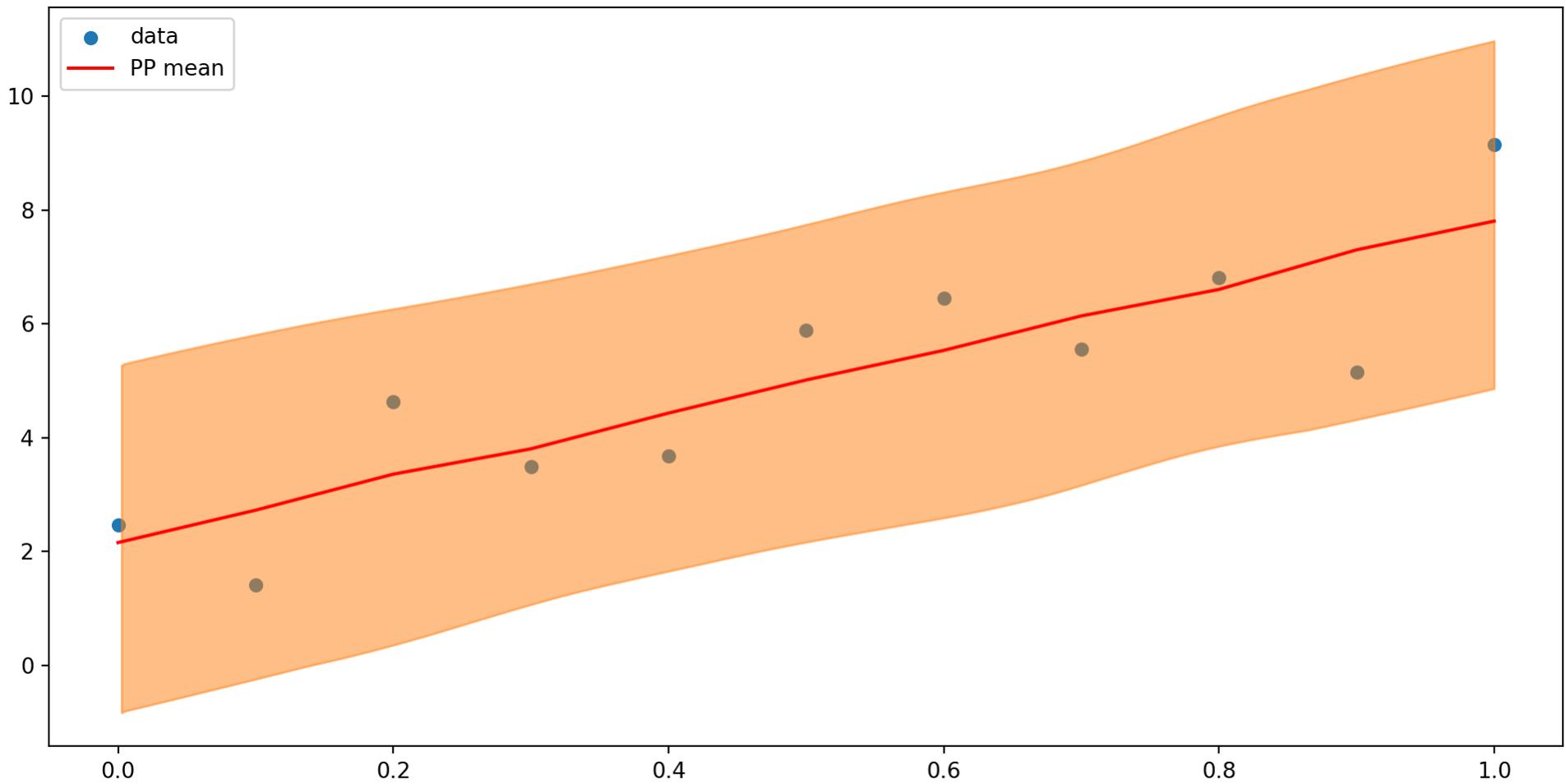
# PP draws

```
1 plt.figure(layout="constrained")
2 plt.scatter(x, y, s=30, label='data')
3 plt.plot(x, pp.posterior_predictive['y'].sel(chain=0).T, c="grey", alpha=0.01)
4 plt.plot(x, np.mean(pp.posterior_predictive['y'].sel(chain=0).T, axis=1), c='red', label="PP mean")
5 plt.legend()
6 plt.show()
```



# PP HDI

```
1 plt.figure(layout="constrained")
2 plt.scatter(x, y, s=30, label='data')
3 plt.plot(x, np.mean(pp.posterior_predictive['y']).sel(chain=0).T, axis=1), c='red', label="PP mean")
4 az.plot_hdi(x, pp.posterior_predictive['y'])
5 plt.legend()
6 plt.show()
```



# Model revision

```
1 with pm.Model() as lm2:
2     m = pm.Normal('m', mu=0, sigma=50)
3     b = pm.Normal('b', mu=0, sigma=50)
4     sigma = pm.HalfNormal('sigma', sigma=5)
5
6     y_hat = pm.Deterministic("y_hat", m*x + b)
7
8     pm.Normal('y', mu=y_hat, sigma=sigma, observed=y)
9
10    trace = pm.sample(random_seed=1234, progressbar=False)
11    pp = pm.sample_posterior_predictive(trace, var_names=["y_hat"], progressbar=False)
```

y

Auto-assigning NUTS sampler...

Initializing NUTS using jitter+adapt\_diag...

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [m, b, sigma]

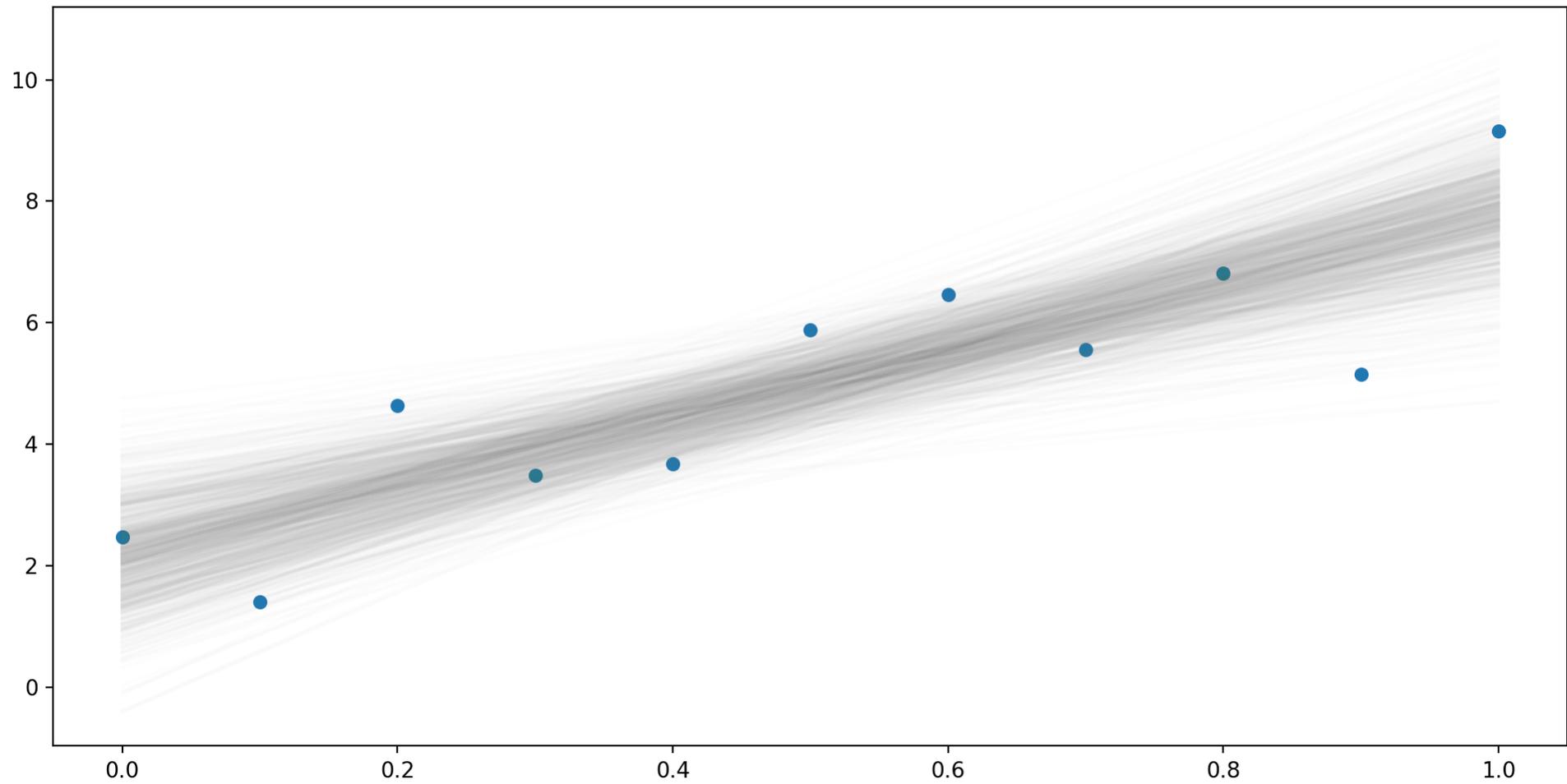
Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws total) took 1 seconds.

Sampling: []

```
1 pm.summary(trace)
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
m	5.573	1.344	2.969	8.042	0.040	0.028	1182.0	1229.0	1.0
b	2.189	0.791	0.659	3.624	0.024	0.018	1106.0	1153.0	1.0
sigma	1.366	0.378	0.788	2.075	0.010	0.007	1452.0	1722.0	1.0
y_hat[0]	2.189	0.791	0.659	3.624	0.024	0.018	1106.0	1153.0	1.0
y_hat[1]	2.746	0.681	1.427	4.009	0.021	0.015	1155.0	1245.0	1.0
y_hat[2]	3.303	0.583	2.228	4.424	0.017	0.012	1254.0	1319.0	1.0
y_hat[3]	3.861	0.501	2.933	4.821	0.013	0.010	1481.0	1786.0	1.0
y_hat[4]	4.418	0.446	3.543	5.218	0.010	0.007	2031.0	2364.0	1.0
y_hat[5]	4.975	0.427	4.187	5.791	0.007	0.005	3266.0	2804.0	1.0
y_hat[6]	5.532	0.449	4.671	6.389	0.007	0.005	4478.0	2810.0	1.0
y_hat[7]	6.090	0.508	5.090	7.017	0.008	0.006	3929.0	2889.0	1.0
y_hat[8]	6.647	0.591	5.568	7.810	0.011	0.008	2963.0	2707.0	1.0
y_hat[9]	7.204	0.691	5.924	8.542	0.014	0.010	2389.0	2337.0	1.0
y_hat[10]	7.761	0.801	6.358	9.405	0.018	0.013	2087.0	2291.0	1.0

```
1 plt.figure(layout="constrained")
2 plt.plot(x, pp.posterior_predictive['y_hat'].sel(chain=0).T, c="grey", alpha=0.01)
3 plt.scatter(x, y, s=30, label='data')
4 plt.show()
```



# Demo 2 - Bayesian Lasso

```
1 n = 50
2 k = 100
3
4 np.random.seed(1234)
5 X = np.random.normal(size=(n, k))
6
7 beta = np.zeros(shape=k)
8 beta[[10, 30, 50, 70]] = 10
9 beta[[20, 40, 60, 80]] = -10
10
11 y = X @ beta + np.random.normal(size=n)
```

# Naive model

```
1 with pm.Model() as bayes_naive:
2     b = pm.Flat("beta", shape=k)
3     s = pm.HalfNormal('sigma', sigma=2)
4
5     pm.Normal("y", mu=X @ b, sigma=s, observed=y)
6
7     trace = pm.sample(progressbar=False, random_seed=12345)
```

y

Auto-assigning NUTS sampler...

Initializing NUTS using jitter+adapt\_diag...

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [beta, sigma]

Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws total) took 70 seconds.

The rhat statistic is larger than 1.01 for some parameters. This indicates problems during sampling. See <https://arxiv.org/abs/1903.08008> for details

The effective sample size per chain is smaller than 100 for some parameters. A higher number is needed for rhat and ess computation. See <https://arxiv.org/abs/1903.08008> for details

There were 113 divergences after tuning. Increase `target\_accept` or reparameterize.

Chain 0 reached the maximum tree depth. Increase `max\_treedepth`, increase `target\_accept` or reparameterize

Chain 1 reached the maximum tree depth. Increase `max\_treedepth`, increase `target\_accept` or reparameterize

Chain 2 reached the maximum tree depth. Increase `max\_treedepth`, increase `target\_accept` or reparameterize

Chain 3 reached the maximum tree depth. Increase `max\_treedepth`, increase `target\_accept` or reparameterize

```
1 az.summary(trace)
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail
beta[0]	643.620	606.139	-622.272	1402.288	261.586	196.065	6.0	14.0
beta[1]	-410.200	341.106	-1078.665	124.358	101.833	76.145	12.0	20.0
beta[2]	717.738	1008.215	-680.198	2520.985	482.446	376.947	5.0	12.0
beta[3]	-519.149	1119.439	-2671.450	1043.628	549.247	419.469	5.0	12.0
beta[4]	1471.100	990.544	-179.163	2808.723	471.027	364.160	5.0	21.0
...	...	...	...	...	...	...	...	...
beta[96]	420.720	1056.327	-1260.449	2382.936	487.813	369.004	5.0	12.0
beta[97]	-143.535	1203.422	-2170.522	1927.823	574.093	436.428	5.0	14.0
beta[98]	-818.080	685.229	-2423.653	190.270	313.078	245.187	6.0	12.0
beta[99]	261.936	912.018	-1070.744	1629.130	440.188	335.261	5.0	15.0
sigma	2.432	1.316	0.461	4.800	0.467	0.343	7.0	27.0

```
[101 rows x 9 columns]
```

# Weakly informative model

```
1 with pm.Model() as bayes_weak:
2     b = pm.Normal("beta", mu=0, sigma=10, shape=k)
3     s = pm.HalfNormal('sigma', sigma=2)
4
5     pm.Normal("y", mu=X @ b, sigma=s, observed=y)
6
7     trace = pm.sample(progressbar=False, random_seed=12345)
```

y

Auto-assigning NUTS sampler...

Initializing NUTS using jitter+adapt\_diag...

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [beta, sigma]

Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws total) took 71 seconds.

The rhat statistic is larger than 1.01 for some parameters. This indicates problems during sampling. See <https://arxiv.org/abs/1903.08008> for details

The effective sample size per chain is smaller than 100 for some parameters. A higher number is needed for rhat and ess computation. See <https://arxiv.org/abs/1903.08008> for details

There were 40 divergences after tuning. Increase `target_accept` or reparameterize.

Chain 2 reached the maximum tree depth. Increase `max_treedepth`, increase `target_accept` or reparameterize

```
1 az.summary(trace)
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
beta[0]	-0.077	6.620	-12.749	12.799	0.200	0.548	830.0	1795.0	1.07
beta[1]	0.527	5.875	-10.243	12.733	0.693	0.525	63.0	2169.0	1.13
beta[2]	2.191	8.614	-11.691	19.642	2.457	1.780	13.0	21.0	1.23
beta[3]	-0.756	6.533	-14.000	10.236	0.958	0.681	43.0	1570.0	1.06
beta[4]	0.399	7.131	-12.064	14.597	1.102	0.785	40.0	1274.0	1.06
...	...	...	...	...	...	...	...	...	...
beta[96]	0.085	6.255	-10.955	12.770	0.510	0.462	162.0	1586.0	1.03
beta[97]	-2.220	6.716	-14.092	11.428	1.059	0.754	40.0	234.0	1.07
beta[98]	-1.101	7.490	-14.253	11.607	2.305	1.677	10.0	31.0	1.29
beta[99]	-1.003	6.763	-14.138	10.319	0.633	0.449	125.0	1170.0	1.02
sigma	1.606	1.172	0.013	3.646	0.393	0.288	7.0	11.0	1.57

```
[101 rows x 9 columns]
```

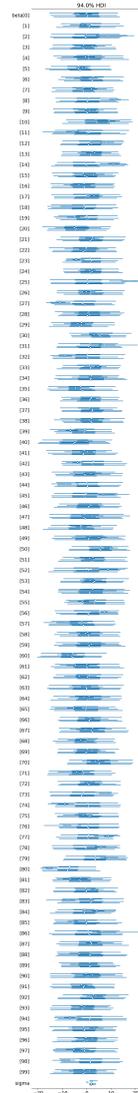
```
1 az.summary(trace).iloc[[10,20,30,40,50,60,70,80]]
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
beta[10]	4.967	6.752	-9.493	15.785	1.221	0.872	31.0	2230.0	1.09
beta[20]	-4.413	6.705	-15.987	8.051	1.157	0.825	34.0	1600.0	1.08
beta[30]	4.986	5.798	-6.198	15.950	0.606	0.560	95.0	1727.0	1.06
beta[40]	-3.742	7.697	-19.201	9.502	1.128	0.802	48.0	263.0	1.05
beta[50]	6.186	6.122	-6.222	16.288	0.635	0.451	122.0	1311.0	1.04
beta[60]	-6.317	6.212	-17.585	6.570	0.327	0.231	364.0	1647.0	1.10
beta[70]	4.856	6.688	-6.852	18.421	0.461	0.423	217.0	1495.0	1.04
beta[80]	-9.648	6.434	-19.370	2.766	1.712	1.256	15.0	217.0	1.18

```

1 ax = az.plot_forest(trace)
2 plt.tight_layout()
3 plt.show()

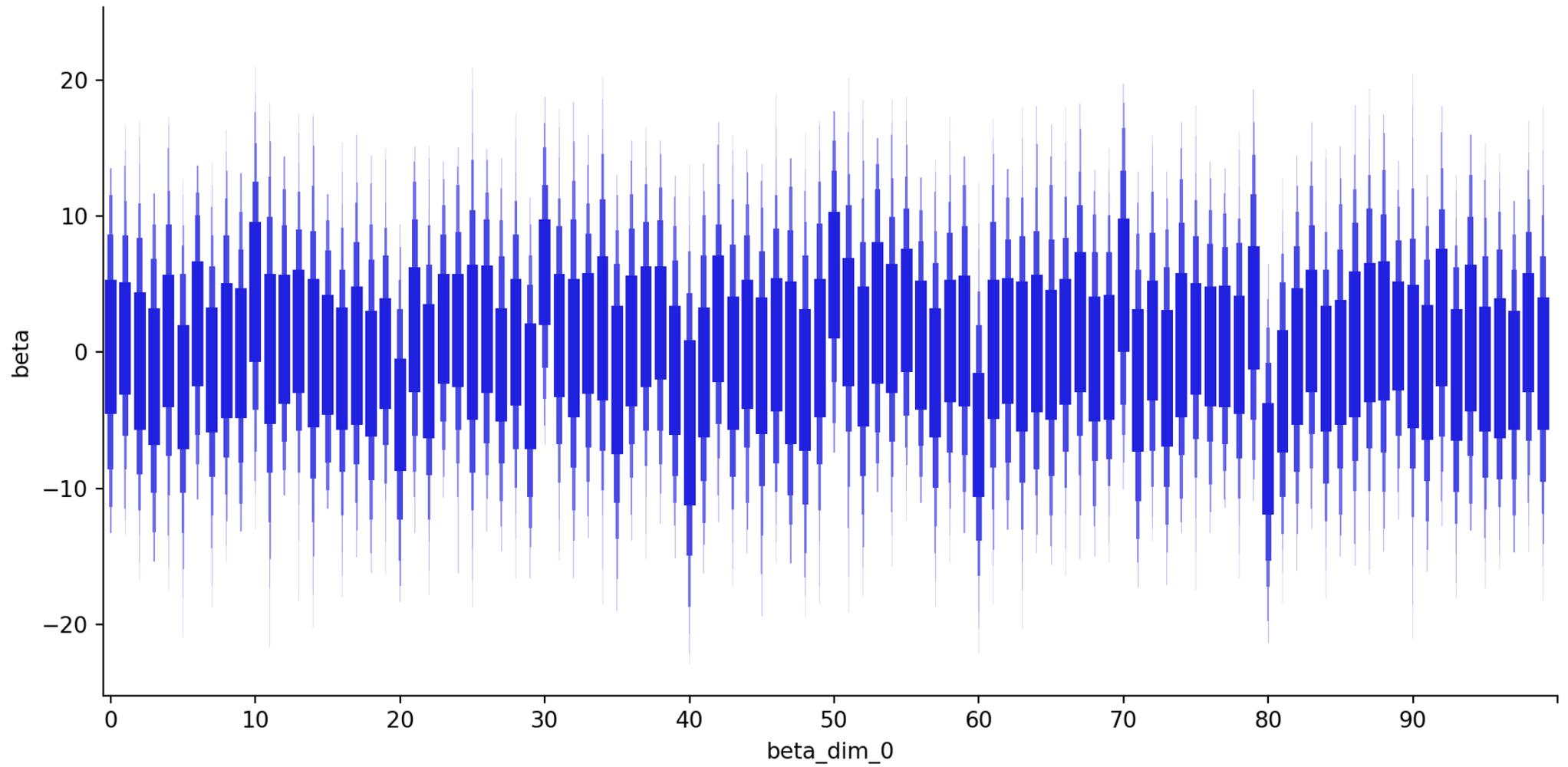
```



# Plot helper

```
1 def plot_slope(trace, prior="beta", chain=0):
2     post = (trace.posterior[prior]
3             .to_dataframe()
4             .reset_index()
5             .query(f"chain == {chain}"))
6
7
8     sns.catplot(x="beta_dim_0", y="beta", data=post, kind="boxen", linewidth=0, color='blue', aspect=2, s
9 plt.tight_layout()
10 plt.xticks(range(0,110,10))
11 plt.show()
12
```

```
1 plot_slope(trace)
```



# Laplace Prior

```
1 with pm.Model() as bayes_lasso:
2     b = pm.Laplace("beta", 0, 1, shape=k)
3     s = pm.HalfNormal('sigma', sigma=1)
4
5     pm.Normal("y", mu=X @ b, sigma=s, observed=y)
6
7     trace = pm.sample(progressbar=False, random_seed=1234)
```

y

Auto-assigning NUTS sampler...

Initializing NUTS using jitter+adapt\_diag...

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [beta, sigma]

Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws total) took 23 seconds.

The rhat statistic is larger than 1.01 for some parameters. This indicates problems during sampling. See <https://arxiv.org/abs/1903.08008> for details

The effective sample size per chain is smaller than 100 for some parameters. A higher number is needed for rhat and ess computation. See <https://arxiv.org/abs/1903.08008> for details

There were 239 divergences after tuning. Increase `target\_accept` or reparameterize.

```
1 az.summary(trace)
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
beta[0]	0.067	0.785	-1.402	1.716	0.011	0.018	4345.0	1972.0	1.02
beta[1]	0.308	0.829	-1.179	1.710	0.152	0.119	38.0	533.0	1.07
beta[2]	-0.045	0.783	-1.634	1.478	0.012	0.015	4101.0	2327.0	1.02
beta[3]	-0.215	0.804	-1.865	1.157	0.075	0.053	95.0	2069.0	1.03
beta[4]	0.063	0.775	-1.508	1.570	0.012	0.013	4381.0	2580.0	1.11
...	...	...	...	...	...	...	...	...	...
beta[96]	0.173	0.774	-1.322	1.427	0.136	0.097	38.0	2541.0	1.07
beta[97]	-0.139	0.721	-1.532	1.351	0.011	0.014	3961.0	2368.0	1.11
beta[98]	0.244	0.708	-1.038	1.689	0.028	0.020	570.0	2040.0	1.01
beta[99]	-0.277	0.750	-1.850	1.061	0.020	0.016	1298.0	1953.0	1.01
sigma	0.825	0.522	0.171	1.790	0.129	0.093	10.0	8.0	1.30

```
[101 rows x 9 columns]
```

```
1 az.summary(trace).iloc[[10,20,30,40,50,60,70,80]]
```

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
beta[10]	8.306	1.184	6.066	10.577	0.025	0.018	2301.0	2176.0	1.04
beta[20]	-8.298	1.253	-10.672	-5.908	0.029	0.021	1860.0	2316.0	1.01
beta[30]	8.673	0.979	6.683	10.340	0.050	0.035	551.0	2361.0	1.01
beta[40]	-8.781	1.517	-11.710	-5.931	0.031	0.022	2408.0	2496.0	1.12
beta[50]	9.076	0.981	7.097	10.773	0.041	0.029	1055.0	2812.0	1.01
beta[60]	-9.313	1.133	-11.377	-7.166	0.133	0.098	69.0	2392.0	1.04
beta[70]	8.636	1.228	6.214	10.546	0.167	0.125	53.0	2107.0	1.05
beta[80]	-9.984	0.888	-11.761	-8.366	0.017	0.012	2629.0	2425.0	1.05

```
1 plot_slope(trace)
```

